

# FUNICULAR FUNNEL SHELLS

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## ABSTRACT

This paper introduces a new typology of structurally efficient, funnel-shaped shells for light and open architectural applications. We discuss a novel approach for structural form finding of funicular shell shapes by defining free boundaries that are balanced by continuous tension rings. Existing funnel-shaped structures illustrate the compelling possibilities for design variation in which vertical and horizontal structural elements transition smoothly, creating exciting spatial configurations. Initially delicate designs are often executed using bulky cantilevers though, which derogate the true elegance of these shapes. The new techniques discussed in this paper demonstrate how the combination of compression-only shells with funicular tension rings leads to a great variety of efficient and expressive forms. The research demonstrates how compression and tension forces can be explicitly controlled and manipulated locally in such structures, using a fully implemented, digital form-finding tool based on Thrust Network Analysis. Thanks to the flexible implementation, resulting shell designs range from simple symmetric shapes to complex, configurations with perforations in the shell and undulating ridge edges. Independent of their complexity, all resulting shells are subject to the clear and comprehensible structural system. The structural system is tested and verified using a 3D-printed, discrete, structural scale model. The elegance and delicate composition of form and force in compression and tension showcased in this funnel-shaped rib-vaulted model hints to the new possibilities for contemporary, architectural applications of the presented structural typology.

**Keywords:** Funicular funnel-shaped design; structural form finding; Thrust Network Analysis; compression shells with tension ties; mushroom structures; digital design tool

## INTRODUCTION

### Spatial Potential of Funnel-shaped Architecture

The term “funnel-shaped”, or “mushroom”, architecture discussed in this paper refers to structures that are supported centrally, forming a cantilevering canopy without columns along the outer edge. Visually, the vertical parts (columns) and the horizontal parts (roof/ceiling) of these structures often merge into one another, forming a continuous space that opens up towards its boundaries. Architectural applications range from large roof structures covering public spaces to smaller interior design projects. These funnel-shaped structures offer great spatial potential. They combine the organic and unique spatial potential of shells and vaults (Chilton 2000), with the openness and lightness associated to modern roof structures in steel and concrete. However, most applications are reduced only to formal considerations, disregarding the structural capacity of these shapes. Evidently, this is true for only decorative ceiling ribs or panels, as in the projects in Figure 1, which can be applied to existing structures to visually change the perception of space.

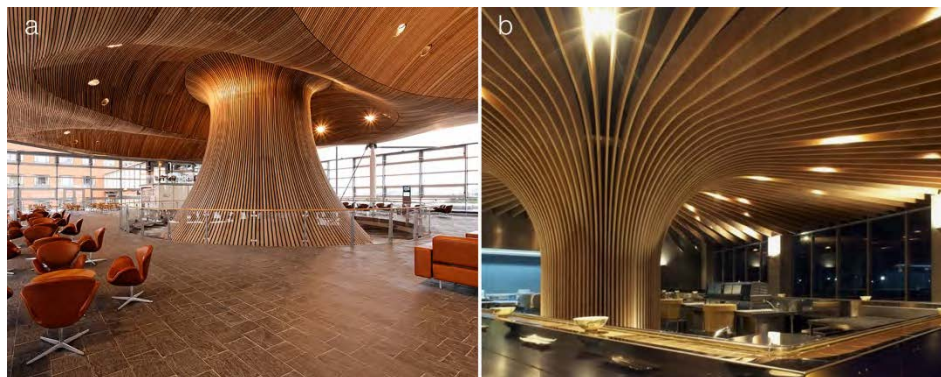


Figure 1: (a) Rogers Stirk Harbour + Partners, National Assembly for Wales (2005) (Photograph: Katsuhisa Kida); (b) Koichi Takada Architects, Restaurant [tree] (2010) (Photograph: Sharrin Rees)

In contrast to these decorative elements, the structures shown in Figure 2 are based on fixed-end columns supporting the cantilevering horizontal elements. For example, a rib mushroomed floor slab construction was designed by *Pier Luigi Nervi* for the *Palazzo del Lavoro* in Turin, Italy (1961), covering a 7900 m<sup>2</sup> exhibition space (Fig 2a). *Frank Lloyd Wright* used connected “lily-pad” columns for the *Johnson Wax Building* in Racine, WI, USA (1936). Yet, these structures are featuring large cantilevering parts, which strongly depend on a relatively high bending capacity of the material and therefore large cross-sections toward the connection with the columns or supports.



Figure 2: (a) Luigi Nervi, Palazzo del Lavoro (1961) (Photograph: Roberto Saba) (b) Frank Lloyd Wright, Johnson Wax Building (1936) (Photograph: Jack Boucher)

Nevertheless, these structures (Figs 1-2) and numerous other examples from medieval times (Fig 3a) show the potential of architectural applications and spatial configurations using funnel geometry (Clifford 2012). However, up to now, the structural potential of this building typology has not been fully exploited in order to extend its architectural vocabulary.

### Structural Potential of Funicular Funnel-shaped Shells

The structural efficiency of shells comes from the fact that their dominated load-bearing is membrane action, i.e. axial compressive or tensile force flows along its center surface, which is inherently more efficient than carrying loads through bending.

Traditional vault construction demonstrates the use of efficient structural form in combination with funnel-shaped architecture, such as the rib vault of the chapel of *Cluny Museum* in Paris, France (1485-1510). This masonry vault of course stands efficiently in compression due to its funicular shape (Fig 3a) resulting in a relatively filigree structure. The contemporary installation *La Voûte de LeFevre* (2012) by *Matter Design* is based on the same structural system, featuring a more irregular geometry thanks to digital, structural form finding and CNC fabrication (Fig 3b).

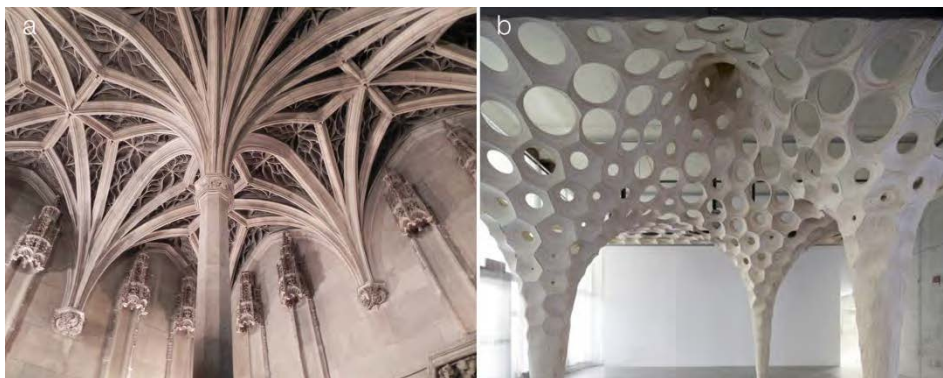


Figure 3: (a) Rib vault of the chapel of Cluny Museum (1485-1510); (b) Matter Design, La Voûte de LeFevre (2012)

Yet, these compression structures depend on fixed boundary conditions, which usually do not allow to open up the sides. The vaults in Figure 3a are stable thanks to massive buttresses taking the horizontal thrust of the rib structure. The openness and lightness of the installation in Figure 3b on the other hand can only be achieved by a steel frame suspended from the ceiling of the gallery space. The structure thus depends on this continuous support, in contrast to the cantilevering structures in Figure 2. The question arises how one can use the structural efficiency of funicular shapes to create open, funnel-shaped shells without the need of a secondary structural frame acting in bending or heavy buttresses.

This paper discusses a direct form-finding approach for funicular funnel shells, and demonstrates the manifold design space of these structures. In the following section, the basic structural principle is illustrated by referring to traditional and contemporary construction techniques. Subsequently, graphic statics and its extension to surface structures, Thrust Network Analysis, are discussed. In the last section, the form-finding approach and its implementation as a design tool are tested through a digital design exploration, and finally verified using a 3D-printed, structural scale model.

## STRUCTURAL PRINCIPLE

The basic structural concept discussed in this paper is based on the use of compression-only vaults in combination with continuous tension ties. This combination is of course known from masonry domes where continuous tension rings are often inserted to resist the tensile hoop force towards the supports. A similar system of combining tension and compression forces was used by Felix Candela for his umbrella shells in the 1950s.

Figure 4 illustrates the use of partial compression arches balanced by tension ties, showing (a) a barrel vault, and (b) a modern concrete arch bridge, both during the erection stage. Both structures are in static equilibrium thanks to temporarily attached cables which counteract the horizontal thrust until the barrel vault, respectively the arch, is closed. In these two examples the tension tie is used as a restraint during construction. However, this temporary state could also represent the structural system of a finalized structure. Hence, these examples serve as a sketch to illustrate the half-arch tension equilibrium system.

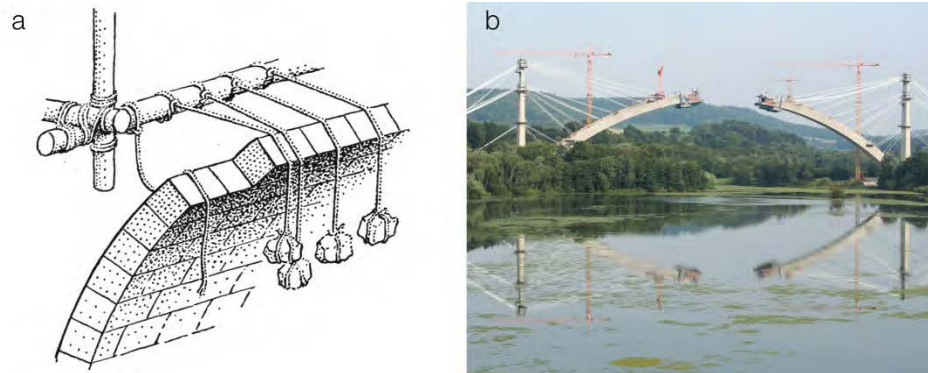


Figure 4: (a) Medieval vault construction using tensioned ropes (Fitchen 1961); (b) construction of the Froschgrundseebrücke, near Coburg, Germany.

## 2D Form Finding using Graphic Statics

The examples in Figure 4 show how funicular structures such as half arches can stand in equilibrium by introducing tension elements. This structural principle to create large cantilevering structures is illustrated in Figure 5 using simple graphic statics. Graphic statics is based on two diagrams: a form diagram, representing the geometry of the pin-jointed structure, and a force diagram, also referred to as (Maxwell-) Cremona diagram, representing the equilibrium of the internal and external forces of the structure (Cremona 1890). The power of graphic statics is based on its inherent bidirectional capabilities; one can either use the form diagram to construct the force diagram, or apply the inverse process and construct parts of the form diagram from an intended force diagram, i.e. either form or force constraints can drive the design exploration (Kilian 2006). Geometrically, the relation between the form and force diagram is called reciprocal (Maxwell 1864). Graphic statics also allows to mutually use compression and tension elements which can be easily identified based on their individual direction in the form and force diagram.

Figure 5 shows the form and force diagram of a funicular arch in compression (a) and the same structure cut in half, with one half flipped horizontally (b). The equilibrium of each node in the form diagram is represented by a closed force vector polygon in the force diagram. Note that the upper ends of both arch segments (b) are in static equilibrium considering the appropriate reaction forces. These reaction forces can either be guaranteed by horizontal supports such as buttresses on both sides or by using a tension tie connecting both ends (Fig 5b).

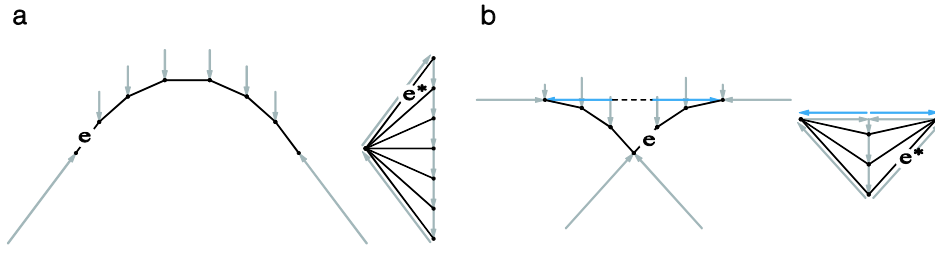


Figure 5: (a) Form and force diagram of a closed funicular arch in compression; (b) two, tied half arches in static equilibrium (tension forces marked in blue)

### 3D Form Finding using extended Thrust Network Analysis

Imagine now that we take several of the tied half arches in Figure 5b, and arrange them in a radial configuration as shown in the equilibrium network  $G$  (Fig 6). Instead of having a tension tie for each pair of aligned arches, this same spatial layout of arches can be balanced by a continuous, polygonal tension ring (Fig 7). The horizontal equilibrium, and hence the plan geometry, of this funicular tension ring is defined by the (equal) thrusts of the half-arches, and can be represented by the reciprocal force diagram  $\Gamma^*$  in Figure 7.

Such reciprocal diagrams of form and forces are used in Thrust Network Analysis (TNA) (Block & Ochsendorf 2007; Block 2009) to explain and control the (horizontal) equilibrium of compression-only vaults. To guarantee compression, Block (2009) explains that corresponding directed edges  $e_i$  and  $e_i^*$  in both diagrams need to have the same directions, i.e. not only be parallel, but furthermore have the same orientation, resulting in positive force densities  $q$  (which are the ratios of the corresponding edge lengths of the force diagram  $\Gamma$  and the form diagram  $\Gamma^*$ ). Tension elements have negative force densities, and their corresponding edges in form and force diagram thus have opposite directions. For two reciprocal diagrams, the sign of axial forces in the elements can thus directly be obtained by using the normalised dot product of the corresponding edges, -1 for tension, and +1 for compression (Eq. 1).

$$q_i = \frac{\|e_i^*\|}{\|e_i\|} \cdot (\hat{e}_i \cdot \hat{e}_i^*) \quad (1)$$

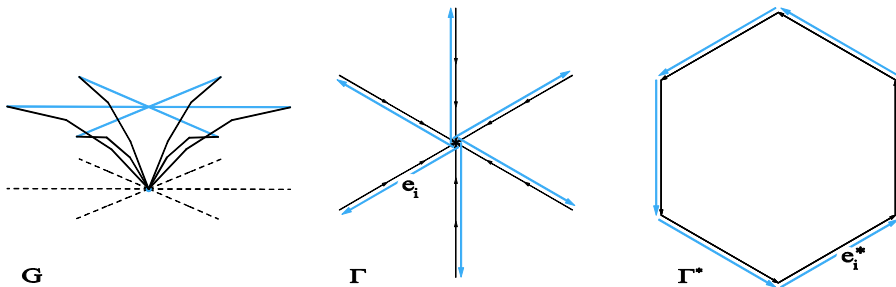


Figure 6: Equilibrium network  $G$ , form diagram  $\Gamma$  and force diagram  $\Gamma^*$  of a radial configuration based on two half arches as shown in Figure 5b (tension elements marked in blue; for illustration purposes overlapping edges are shown offset to each other)

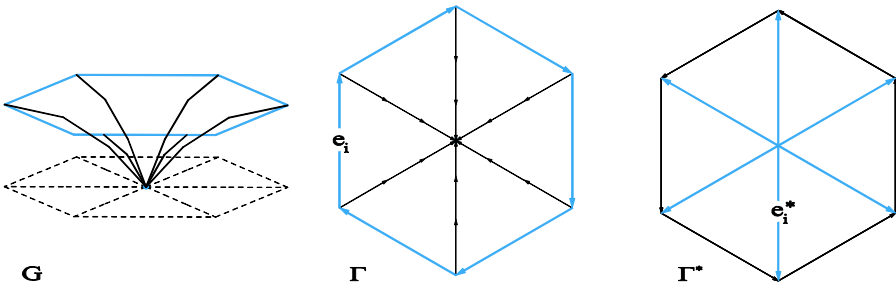


Figure 7: Equilibrium network  $G$ , form diagram  $\Gamma$  and force diagram  $\Gamma^*$  of a radial configuration with a continuous tension tie based on the configuration shown in Figure 6 (tension elements marked in blue)



Figure 8 shows the same configuration in plan as in Figure 7, but with inverted support conditions. While the inner node is free to move vertically, the nodes along the outer boundary are fixed in the vertical direction. Note that this results in a simple dome structure with a continuous tension tie resisting the tensile hoop force at the supports.

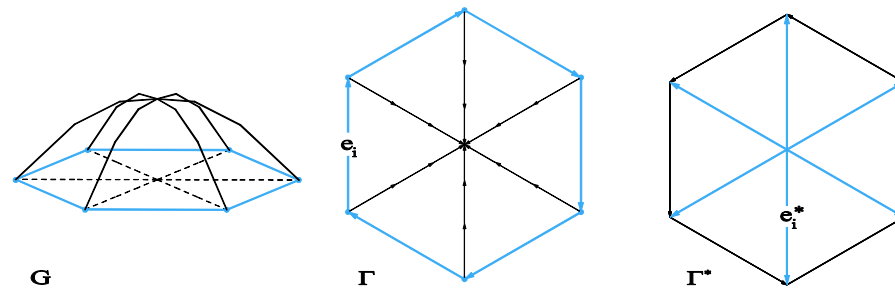


Figure 8: Equilibrium network  $G$ , form diagram  $\Gamma$  and force diagram  $\Gamma^*$  of a radial configuration with a continuous tension tie resisting the tensile hoop force at the supports (tension elements marked in blue)

## RESULTS

In this section, the form-finding approach and its implementation as a design tool is tested through a digital design exploration, and is finally verified using a 3D-printed structural scale model.

### Design Tool Implementation

To explore the design space of funicular funnel shells, the TNA method was implemented as an interactive, digital tool. It takes advantage of the inherent, bidirectional interdependency of form and forces represented in visual diagrams, which are essential for a user-driven and controlled exploration in the structural form-finding process. Thus, the implementation and design of the form-finding tool focused on design through exploration, underlining the visual and playful nature of the approach, mainly targeting the early structural design phases. The software was developed for in-house research but also released as simple design tool for compression-only structures, so without the option to include the continuous tension ties, under the name RhinoVAULT (Rippmann et al. 2012) as a free plug-in for the CAD software Rhinoceros (McNeel 2013).

### Design Exploration

Figures 9-11 show the design exploration of various funicular funnel shells generated with the tool. The variations of equilibrium networks in Figure 9 are realized by simply changing the definition of free or fixed support nodes (the latter marked with blue dots). The only other modification of the diagrams is the uniform scaling of the edges of the force diagrams. The force diagrams are drawn to the same scale so that the overall magnitude of the horizontal thrusts in each structure can be visually compared. Based on these simple modifications during the design exploration process, the resulting funicular funnel shells (a-f) vary greatly in shape and spatial articulation.

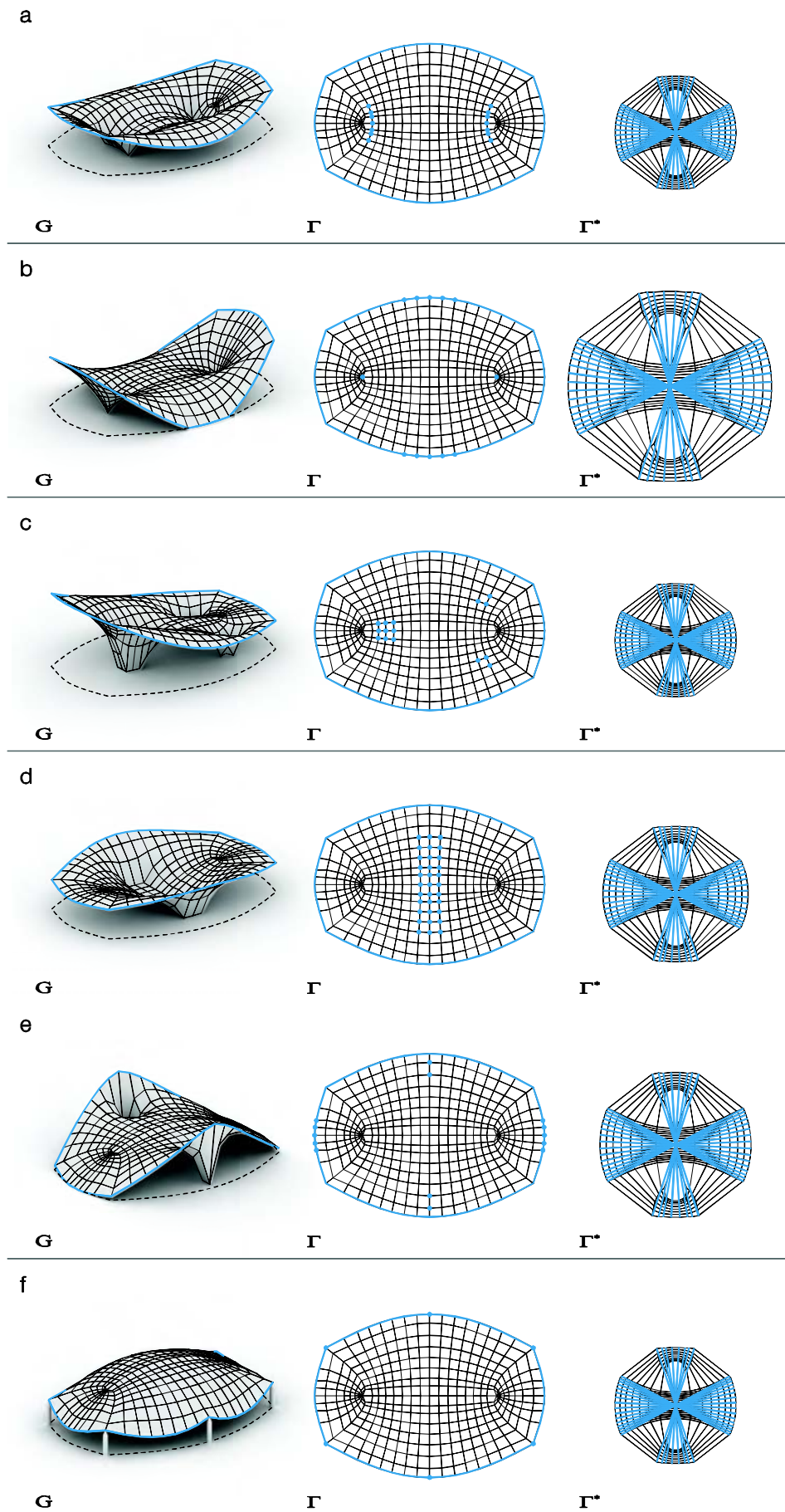


Figure 9: Design exploration of various funicular funnel shells (a-f) (the tension elements are highlighted in blue) by changing the definition of free or fixed support nodes (marked with blue dots) and the overall magnitude of the horizontal thrusts in each structure

The integration of holes in the shell surface as shown in Figure 10 demands the topological modification of the form diagram. These openings always form a funicular polygon in the form diagram and are by definition convex; their force equilibrium in the force diagram has a star-shaped topology (Rippmann & Block 2013). The cantilevering ridge edge can be seen as an inverted opening forming a convex boundary.

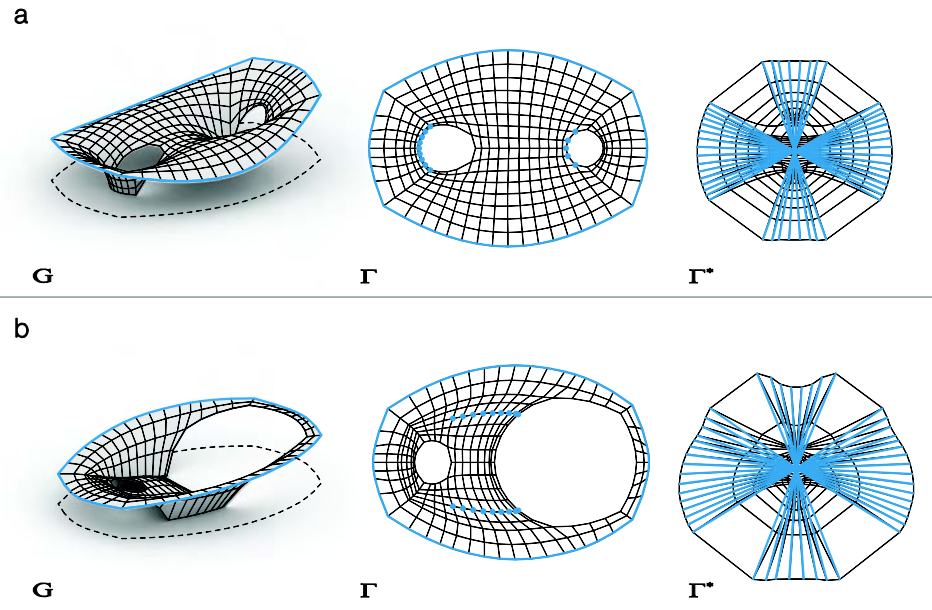


Figure 10: Design exploration of various funicular funnel shells with openings

Further, the TNA framework allows controlling the multiple degrees of freedom in statically indeterminate networks. In other words, a statically indeterminate form or force diagram can be geometrically modified while keeping horizontal equilibrium. This means that the length of corresponding elements of the form and force diagram can be modified while guaranteeing their parallel configuration and direction. Consequently, this leads to a local or global increase or decrease of forces since the length of each element in the force diagram represents the horizontal force component of the corresponding element in the structure. In Figure 11, the force diagram in (b) shows the local attraction of horizontal thrust towards the centre of the structure resulting in a local compression ring and hence a crease in thrust network. It is important to note though that, in contrast to compression shells with fixed supports, the freedom to change the force distribution of structures with continuous tension ties is limited due to the geometrically much more strongly constrained force diagram.

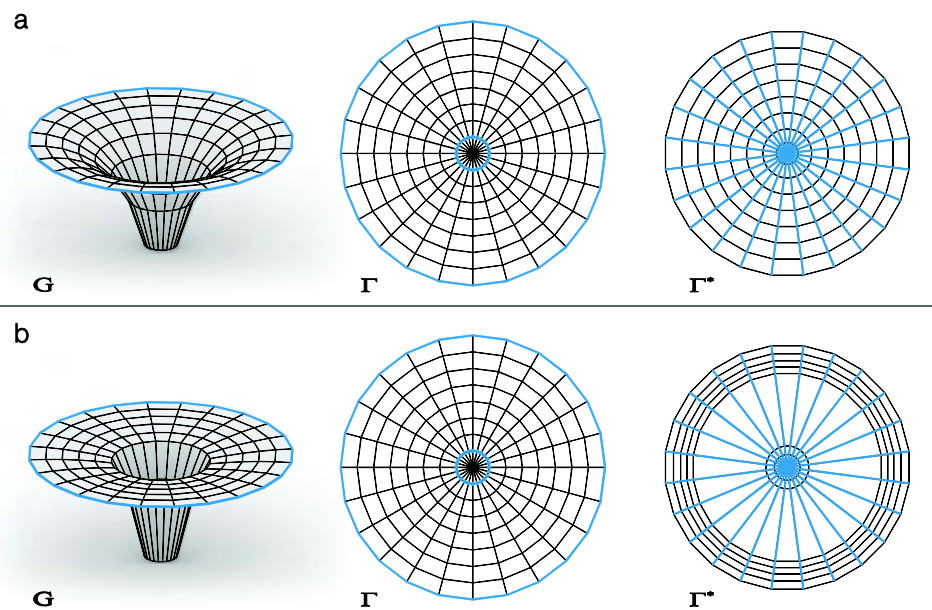


Figure 11: A different force distribution for the same form diagram  $\Gamma$  in (a) and results in a crease in the equilibrium network  $G$  of (b)

### Funicular Funnel Rib Vault

In general, the equilibrium stability can effectively be tested with scale models (Van Mele et al., 2012). The presented structural system was tested and verified using a 3D-printed, discrete, structural scale model (Fig 12) to validate the digital results of the form finding implementation with the physical model. The scale model is made out of discrete (unglued and mechanically not connected), 3D-printed pieces; it is not a structure by itself, but rather an extreme structural model as the pin-jointed, mechanically not connected pieces have a non-rigid (not triangulated) topology. If the solution would not act in pure compression equilibrium (and tension along the tension tie), it would immediately become unstable and collapse.



Figure 12: Structural scale model of a discrete funnel-shaped rib vault

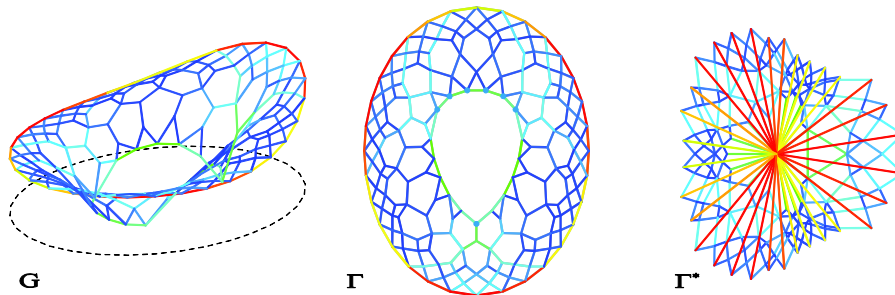


Figure 13: Equilibrium network  $G$ , form diagram  $\Gamma$  and force diagram  $\Gamma^*$  of the discrete funnel-shaped rib vault. The colours are directly related to the length of the individual edges in the force diagram and hence visualize the magnitudes of horizontal thrust

As demonstrated by the collapse sequence in Figure 14, the 3D-printed model is an extreme structural model, as it is made out of discrete, unglued pieces, forming unstable mechanisms, which can only be balanced if the ribs are in perfect axial, compression. The sudden collapse, when cutting the tie, is a demonstration of the structural honesty of the model. As for tree structures or any result of form finding in general, form-found to act in compression only for a defined, dominant loading case (Lachauer & Block, 2012), this type of structures need to be realized with stiff nodes in order to resist live, non-funicular loading cases, which can be defined and dimensioned through the analysis of a materialized, form-found shape.



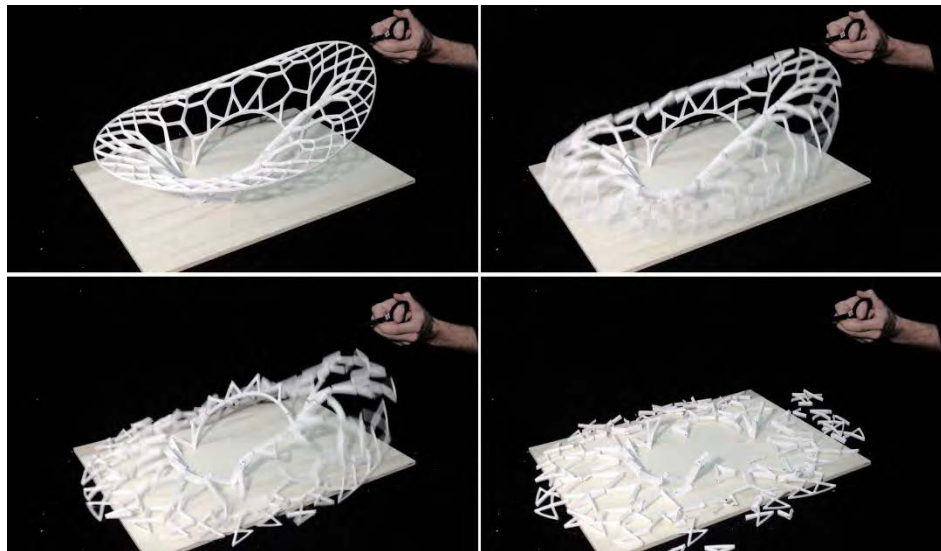


Figure 14: Collapse of the 3D-printed funnel-shaped rib vault by cutting the continuous tension tie

## CONCLUSION AND FURTHER RESEARCH

This paper discussed how funnel geometries can be made structurally efficient, as a combination of a three-dimensional equivalent of funicular half-arches balanced by tension ties. It was shown how Thrust Network Analysis can be extended to incorporate tension elements, using directed elements in the form and force diagram, to create continuous tension rings. These concepts have been implemented by extending RhinoVAULT to these new boundary conditions. A simple design exploration showed the variety of possible shapes using only simple modification strategies. Lastly, a structural model was developed to validate the equilibrium solutions generated with the approach. This model furthermore hints at the attractive possibilities of filigree ribbed funnel structures, reminiscent of Schlaich's beautiful tree structures.

Although the current, direct implementation allows an intuitive exploration of funicular funnel shells, as clear from the exploration in the result section, the following objectives represent routes for further research:

- Understand the actual dependencies and constraints of the form and force diagram for this new type of boundary condition better to fully explore the possibilities of the funicular funnel shell typology;
- Include feedback, or immediately constraints, on the global moment equilibrium during the form-finding process: shell whose centres of gravity fall outside of the convex hull of the supports, can of course not stand as a combined pure compression shell and tension rings; and
- Formulate the form finding as a best-fit (to a target surface/geometry) optimization problem, as in Block & Lachauer (2011) or Panozzo et al. (2013) for compression-only shells.

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