Form finding and structural analysis of a freeform stone vault

Tom VAN MELE*, Anjali MEHROTRA[b], Tomás MENDEZ ECHENAGUCIA[a], Ursula FRICK[a],
John OCHSENDORF[c,d], Matthew DEJONG[b,d], Philippe BLOCK[a,d]

*ETH Zurich, Institute of Technology in Architecture, Block Research Group
Stefano-Franscini-Platz 5, HIL H 46.2, 8093 Zurich, Switzerland
van.mele@arch.ethz.ch

[a] ETH Zurich, Institute of Technology in Architecture, Block Research Group, Zurich, Switzerland
[b] University of Cambridge, Department of Engineering, Cambridge, UK
[c] Massachusetts Institute of Technology, Department of Architecture - Department of Civil and Environmental Engineering, Cambridge, MA, USA
[d] Ochsendorf DeJong & Block, Cambridge, MA, USA

Abstract

This paper provides an overview of the form finding and analysis of a freeform, unreinforced, cut-stone, dry-set vault with 399 voussoirs. The vault covers an area of 75 m^2 and spans more than 15 m in multiple directions. The thickness of the discrete, stone shell varies from 12 cm at the supports to 5 cm at the highest points. The paper describes the TNA-based funicular form-finding process starting from a geometrically defined target surface and a layout of the horizontal thrust in the system, and resulting in a three-dimensional network of forces in compression for the given loading and boundary conditions. Furthermore, the paper discusses the discretisation of the funicular geometry into voussoirs, and describes the conversion of the complex voussoir geometry into tetrahedralised compound blocks for analysis in 3DEC. Finally, the results of the structural analysis addressing both static and dynamic loading conditions are discussed.

Keywords: discrete structures, stone masonry, thrust network analysis, discrete element modelling

1. Introduction

This paper describes the form-finding process and equilibrium analysis of the Armadillo Vault, an unreinforced, cut-stone vault constructed in the Corderie dell'Arsenale for the 15th International Architecture Exhibition - la Biennale di Venezia, curated by Alejandro Aravena. The vault is the centrepiece of the "Beyond Bending" exhibition, which advocates for the logic of compression-only forms, not only because of their uniquely expressive aesthetics, but also because of their potential to achieve efficiency and stability through geometry. The doubly curved vault consists of 399 individual limestone blocks or *voussoirs* that are assembled without mortar or other structural connections. The vault thus stands in pure compression and covers an area of 75 m^2, with free spans of more than 15 m. The structure has a (more or less) triangular shape in plan, with three linear supports along the boundary and one support in the middle. The unsupported edges between the boundary supports create openings that provide access to the space underneath. The existing columns in the exhibition space pierce through the structure's surface through two large openings, one of which is partially supported (Figure 1). The supports are made of 20mm-thick steel plates, and designed to distribute the weight of the vault evenly over a large enough area to reduce the load on the floor of the protected building to the prescribed limitation of 600 kg/m^2. A system of steel ties connects the supports (see Figure 1) and absorbs the horizontal thrust. The ties are necessary because anchoring was not allowed. They are left exposed to visualise how the vault actually stands and to emphasise that in these constrained conditions the stone structure would not be stable without them.

Figure 1. The Armadillo Vault in the Corderie dell'Arsenale at the 2016 Architecture Biennale in Venice, Italy.
Photos: Iwan Baan

The design of an unreinforced, discrete, cut-stone, dry-set vault with complex geometry is a complicated process that requires an integrated computational setup that from the early design and form-finding stages accounts for structural and architectural considerations, and fabrication and assembly constraints. In this paper, we focus on the aspects of this process related to the structural design and analysis of the vault. For details about the architectural geometry and the fabrication and assembly processes, see Rippmann et al. [1]. In Section 2, the process of finding an appropriate form that allows the vault to stand in pure compression, taking into account the architectural and functional requirements of the object in relation to the exhibition space, is described. Section 3 discusses how the geometry of the individual voussoirs is designed based on the thrust surface and the corresponding weight distribution, and a description of how the voussoir geometry is converted to a discrete element model and analysed under different loading conditions.

2. Form Finding

The funicular shape of the vault is the result of a form-finding process based on Thrust Network Analysis (Block and Ochsendorf [2]). As a first step, preliminary design alternatives were sketched using RhinoVAULT (Rippmann et al. [3]). From the chosen sketch design, a control mesh was created and used to iteratively refine the geometry to increase double curvature and distribute the vertical reaction forces as evenly as possible over the supports. For each of these geometrically designed target shapes, a "best-fit" procedure was used to find the closest possible network of compressive forces under the given loads (Van Mele et al. [4]). Finally, the resulting thrust network was converted to a dense, smooth mesh representing the middle surface of the stone envelope of the vault.

2.1. Target Geometry

The designed geometry of the vault is more or less triangular in plan. There are two large openings through which the existing columns of the Corderie can penetrate the surface of the vault. One of the openings is partially supported in the centre of the covered area at support D (Figure 2). On the boundary, there are two large linear supports, B and C, and one more concentrated support, A. All supports are at ground level. The "pulldown" at support D not only creates a more interesting visual experience, but also helps create more curvature and therefore a more robust geometry.

(a)   (b)
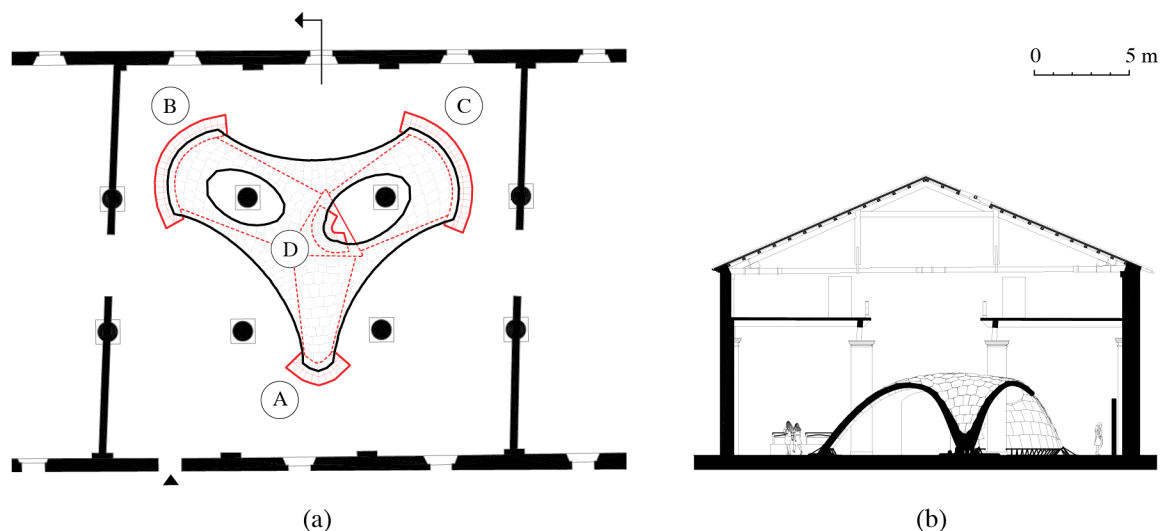
Figure 2. (a) Plan view of the vault indicating the different supports and the system of tension ties. (b) Section through the Corderie dell'Arsenale.

2.2. Form Diagram

Thrust Network Analysis (TNA) uses form and force diagrams to control the spatial equilibrium of a three-dimensional network of compressive forces for a given set of parallel loads. The form diagram represents the directions along which horizontal forces are allowed to flow. The magnitude of forces along those directions is represented by the force diagram.

The layout of force directions was derived from the geometric and structural features of the three-dimensional target geometry. First, a low-poly mesh identifies the different patches with clear force directions. Singularities are formed where the patches meet (Figure 3). The patches are converted to quad meshes according to the imagined principal force directions. The quads are triangulated to provide additional degrees of freedom to the force flow.

(a)   (b)   (c)
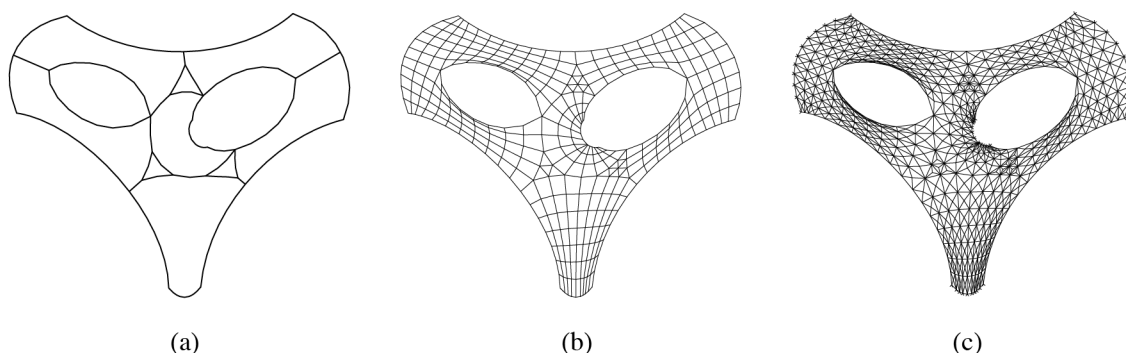
Figure 3. (a) Discretisation of the vault into patches associated with the different supports, and resulting singularities. (b) Quad mesh layout per patch. (c) Triangulation of the pattern for additional degrees of freedom.

2.3. Self-weight

The dominant loading on this vault is its own weight. During the form-finding process only this load is taken into account to define the geometry of the middle surface of the vault. The intrados and extrados are offset from that middle surface according to the local thickness defining the boundaries of the stone envelope. This thickness may be increased to accommodate the development of thrust networks within the middle third of the envelope for all other loading cases.

Because of the prescribed maximum stress on the floor of the exhibition space (600 kg/m^2), the weight of the stone shell had to be reduced to the absolute minimum. Through a series of material tests, the minimum required thickness of the voussoirs to avoid spalling of the stone due to eccentric loading at the interfaces, was determined to be 5 cm. For both structural and aesthetic reasons, the thickness was increased at the supports: 8 cm at supports B and C, and 12 cm at supports A and D. From these locally prescribed thicknesses, a smooth distribution of thickness over the vertices of the form diagram was computed using an interpolation algorithm.

2.4. Thrust Surface

The best-fit algorithm was used to find the specific distribution of forces along the predefined thrust pattern represented by the form diagram that maps the three-dimensional network as close as possible to the geometric target. The force diagram depicted in Figure 4 is the final distribution. Note that the target geometry was updated several times during this optimisation process to increase local double curvature where needed, and distribute the reaction forces more evenly over the supports.

Figure 4 furthermore visualises the distribution of horizontal forces with proportional thicknesses of the form diagram, and depicts a lumped stress distribution. The lumped stresses are computed by averaging in each node the forces in the connected edges divided by their tributary width and the local thickness. It shows that stresses are extremely low and do not exceed 0.1 MPa, which is two orders of magnitude below the compressive strength of approximately 22 MPa of the Cedar Hill limestone used.
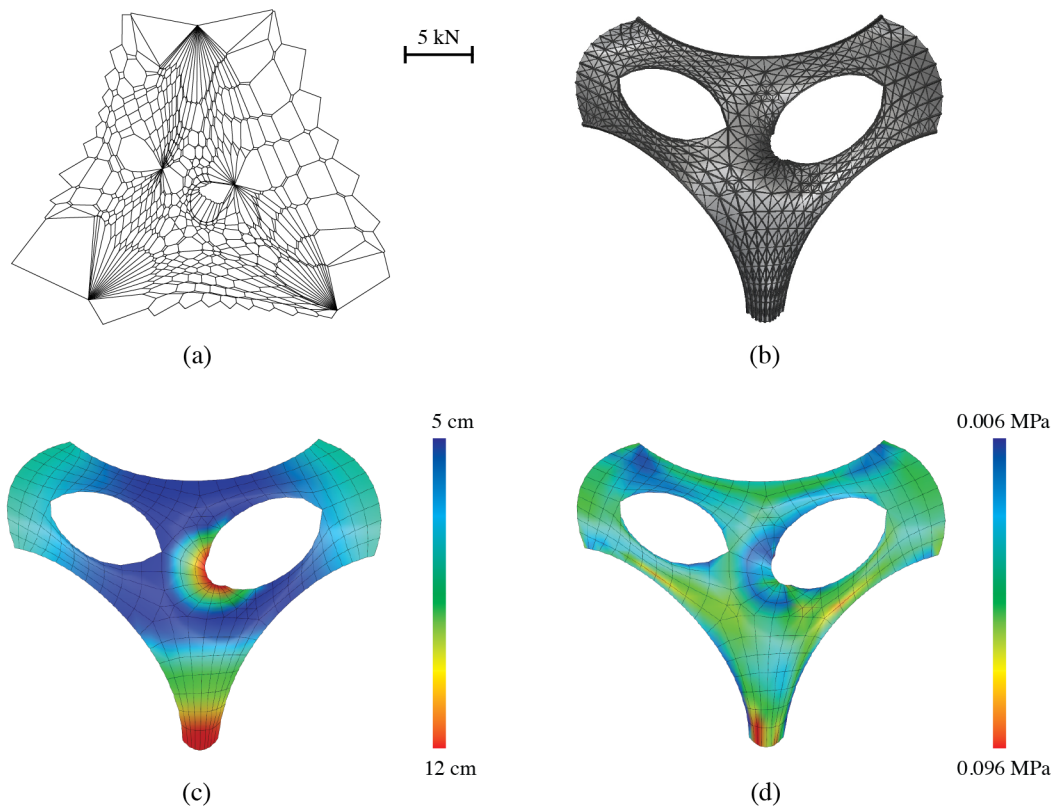

Figure 4. (a) Force diagram representing the "best-fitting" distribution of horizontal forces. (b) Forces in the edges of the thrust network, with line thicknesses indicating relative magnitudes. (c) Stone thickness distribution. (d) Lumped stress distribution.

Figure 5 gives an overview of the reaction forces. For comparison, the reaction forces have been lumped to one resultant per support. Note that the horizontal thrust is spread evenly over the supports at the boundary. The horizontal reaction at the central support is almost zero. The supports on the boundary are connected by a system of ties that absorbs the outward thrust of the vault. Each support has two ties, which are connected to a triangular frame around (but not connected to) the central support (Figure 2). Only the vertical components of the reaction forces are thus transferred to the ground. These vertical reaction forces are distributed more or less evenly over all four supports (A = -40.2 kN, B = -45.5 kN, C = -50.9 kN and D = -53.7 kN), with a slight accumulation at support D.
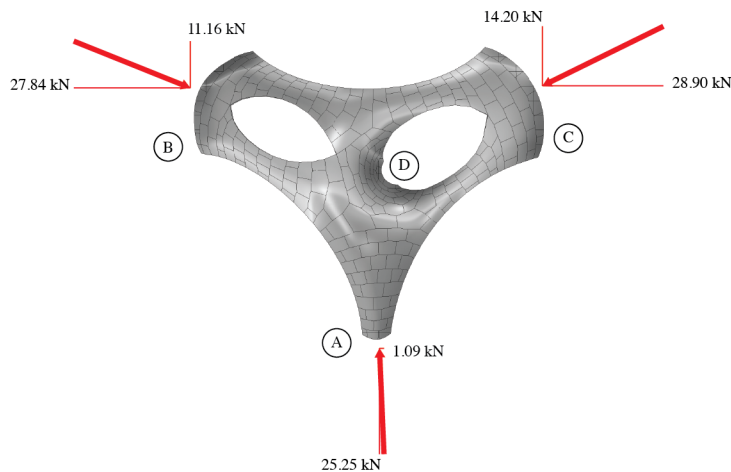
Figure 5. Overview of the horizontal thrust.

3. Equilibrium Analysis

In this section, the discretisation of the stone envelope into individual voussoirs is described, as well as the equilibrium analysis of this discrete-element model using 3DEC for the dominant loading case (self-weight). Results from other loading cases, including differential settlements at the supports, point loads on the unsupported edges, and lateral earthquake loads, are also presented.

3.1. Tessellation

The stone envelope defined by the intrados and extrados surfaces is discretised into voussoirs following a tessellation pattern taking into account the fabrication and assembly requirements. First, the stone surface is divided into courses (Figure 6a). These are initially formed by geodesic lines and then optimised to be as perpendicular as possible to the flow of forces towards the supports. The tessellation mesh then divides the courses into voussoirs, in a staggered arrangement from one course to the next (Figure 6b). Finally, all faces of the tessellation mesh are forced to be convex to simplify the fabrication process.
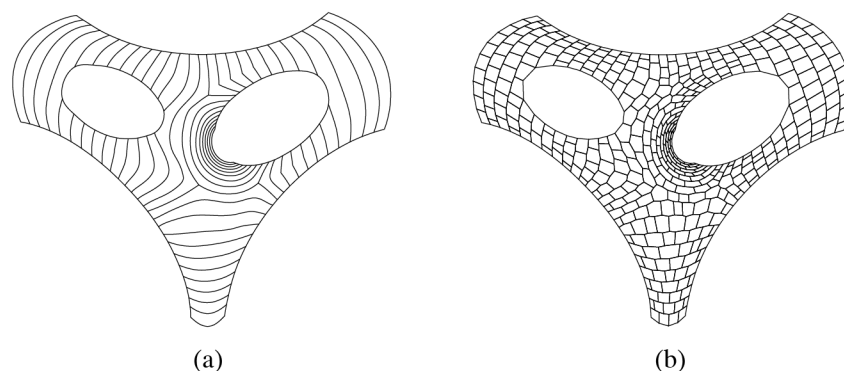
<div align="center">(a)   (b)</div>

Figure 6. (a) Course lines. (b) Staggered tessellation pattern.

3.2. Voussoir Geometry and Discretisation

The geometry of the voussoirs is a direct result of fabrication constraints and structural requirements (Rippmann et al. [1]). The extrados of each voussoir is flat (Figure 7a) and used as a base for further processing of the other surfaces. The intrados is formed by parallel cuts with a circular blade that are spaced such that thin stone fins remain. The fins are hammered off manually to create a rough, but curved surface (Figure 7c). The side surfaces that more closely align with the direction of the course lines (Figure 6a) are the main load-transferring interfaces. These interfaces are processed with custom profiling tools that create ruled surfaces with male/female notches (figure 7d). The notches are primarily used as registration marks to simplify assembly. They also add some shear capacity to the assembly and therefore provide an additional safety against sliding failure. The other side surfaces are created with simple, planar cuts (Figure 7e).

For the equilibrium analysis with 3DEC, all external surfaces of the voussoirs are triangulated and their interior discretised into tetrahedrons corresponding to the external triangulation. For simplicity, the registration notches were not included in the discretised analysis model.
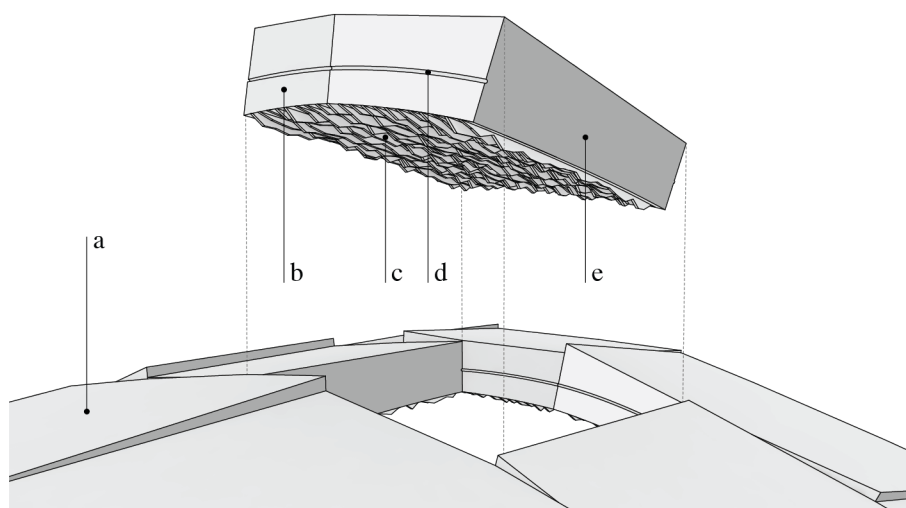

Figure 7. Overview of voussoir geometry: a) flat extrados surface, b) ruled interface perpendicular to the primary loading direction, c) rough-cut, curved intrados surface, d) registration notch, e) flat interface parallel to the primary loading direction.

Note that the surface of the vault has several areas with negative gaussian curvature. It is not possible to fit a mesh with convex, flat faces onto such a surface (Li et al. [5]). Therefore, the faces of the extrados were allowed to disconnect and create a stepped exterior. As a consequence, the side surfaces of adjacent voussoirs are only partly in contact with each other. The actual overlaps between the interfaces were discretised separately to prevent overlaps between the volumes of the voussoirs in those areas due to otherwise mismatching discretisations.

3.4. Analysis Model

Upon obtaining the final discretised voussoir geometry, the resulting Rhino model was used to generate the corresponding discrete element model for analysis in 3DEC. Rigid blocks with a density of 2320 kg/m^3 (mass density of cream limestone) were used in the analysis. The deformability of both the blocks and the joints was accounted for when defining the joint stiffness. To reduce computation time, artificially low values of the joint shear and normal stiffnesses were specified (2.77x106 kPa/m and 1.66x106 kPa/m, respectively). These values are approximately 25 times lower than the actual calculated stiffnesses. A friction angle of 35° at all block contacts was assumed. To be conservative, all joint contacts were modelled as planar, though the majority of joints in the actual structure have shear keys for registration and to prevent sliding.

3.5. Load Cases

Initially, gravity was applied to the discrete element model and an equilibrium solution was found. The vertical reactions and horizontal thrust values at all supports were found to be within 50% and 75% of the values found using TNA, respectively. Based on the location of the maximum vertical displacements recorded under this loading, and based on intuition, numerous additional load cases were considered. In this paper, the following load cases were applied (Figure 8) :

1. Point load (PL) applied to a single block at an unsupported edge of the vault in 20 kg increments.
2. Concentrated loads (CL1 and CL2) applied to 2 or 3 blocks at unsupported edges in 100 kg increments.
3. Line load (LL) applied to a strip of 3 blocks in 200 kg increments.
4. Lateral earthquake loads in the form of constant horizontal accelerations applied in the two weakest directions (Hor. 1 and Hor. 2) in 0.025 g increments.
5. Vertical support displacement (SD) applied at support D in 2.5 mm increments, with a maximum displacement of 10 mm.
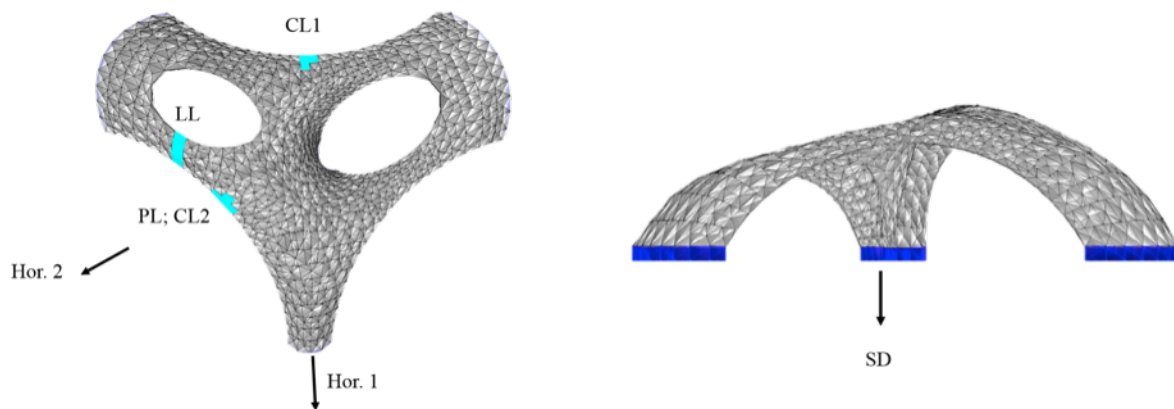

Figure 8. Overview of the load cases applied to vault.

3.6. Results

Under the influence of gravity alone, a maximum vertical displacement of 3.1 mm was found in the area of the vault between supports B and C, while a second slightly smaller displacement of 2.6 mm was recorded between supports A and B. The location and distribution of these vertical displacements are illustrated by the contour plot in Figure 9. Note that the actual expected vertical displacement of the real vault would be smaller than this due to the artificially low joint stiffness specified.
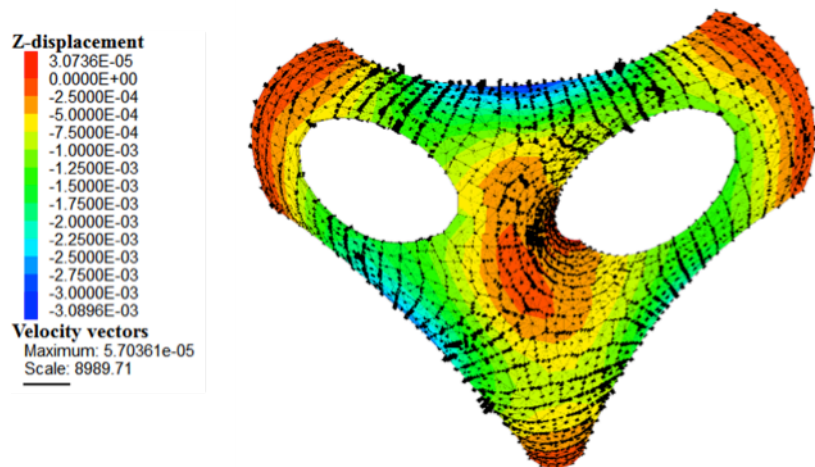

Figure 9. Variation of vertical displacement of the vault under gravity alone.

In the case of the point load (PL), local failure of the vault occurred under a weight of 1300 kg. Failure consisted of slippage of both the block to which the load was applied as well as two adjacent edge blocks. The concentrated load case (CL2) was applied to these same three blocks, with failure in this case taking place under a similar 1400 kg of loading. However, the local failure mechanism was much larger, and several adjacent edge blocks were found to slip as well. Similarly, for the concentrated load case CL1, local collapse occurs for 1000 kg of loading in the form of slippage of the loaded blocks as well as two adjacent edge blocks. For the line load (LL), failure occurred under a total weight of 1400 kg, in the form of a very pronounced local mechanism as depicted in Figure 10a. Thus for the vault to fail through any of these load cases, 12-17 adults (assuming an average weight of 85 kg) would have to be standing simultaneously on 1-3 adjacent blocks.

In the case of the lateral earthquake loads (Hor. 1 and Hor. 2), failure was found to occur in the form of global collapse mechanisms for constant horizontal accelerations of 0.375g and 0.30g respectively. Figure 10b illustrates the global collapse mechanism for the Hor. 1 load case. For the 10.0 mm vertical displacement (SD) of support D, a maximum vertical deflection of 11.6 mm occurred (not shown here). This displacement was concentrated, for the most part, near support D itself. Furthermore, a maximum joint opening of 0.9 mm was observed at the opening of the vault near support C – which is negligible when compared to the overall size and span of the structure.

Note that an imperfect geometry or local compressive failure at imperfect joint contacts could cause these failure loads to decrease, or the maximum displacement values to increase. This is another reason that precise control of the geometry during construction is important. However, the analysis is in another sense conservative because the joint stiffness was reduced, allowing snap through failure to occur more easily, and the presence of shear keys in the real structure, which would certainly increase sliding resistance, were neglected in the simulation.
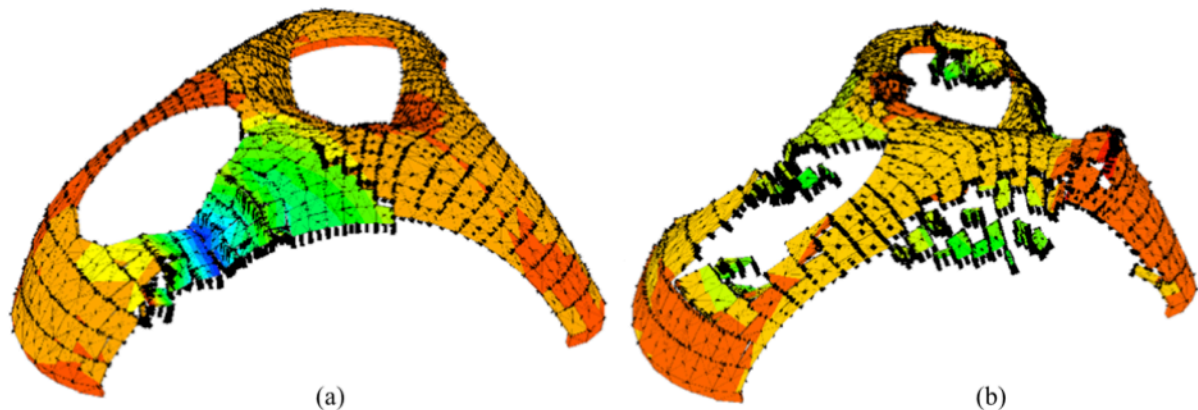
Figure 10. (a) Line load local failure mechanism. (b) Lateral earthquake load (Hor. 1) global failure mechanism.

4. Discussion and summary

This paper gave an overview of the non-typical structural design and analysis methods used for the development of a unique, unreinforced, cut-stone vault that is proportionally thinner than an eggshell. The funicular shape of the vault was designed using a form-finding process based on Thrust Network Analysis. The thrust surface was discretised into voussoirs according to a tessellation pattern and taking into account fabrication constraints, structural requirements, and architectural considerations. A discrete-element model of the assembly of 399 individual voussoirs was used for equilibrium analysis under different loading conditions. Without these geometry and equilibrium-based methods and an integrated computational setup this project would not have been possible.

Acknowledgements

References

[1] Rippmann M., Van Mele T., Popescu M., Augustynowicz E., Méndez Echenagucia T., Calvo Barentin C., Frick U. and Block P., The Armadillo Vault: Computational design and digital fabrication of a freeform stone shell, *Advances in Architectural Geometry*, 2016.

[2] Block, P. & Ochsendorf, J., Thrust network analysis: a new methodology for three-dimensional equilibrium, *Journal of the international association for shell and spatial structures*, 2007, 48(3), pp 1-8.

[3] Rippmann M., Lachauer L. and Block P., Interactive Vault Design, *International Journal of Space Structures*, 2012, 27(4): pp 219-230, Multi-Science Publishing.

[4] Van Mele T., Panozzo D., Sorkine-Hornung O. and Block P., Best-fit Thrust Network Analysis - Rationalization of freeform meshes. In S. Adriaenssens, P. Block, D. Veenendaal, C. Williams (editors) *Shell Structures for Architecture: Form Finding and Optimization*, 2014.

[5] Li, Y., Liu, Y. & Wang, W., Planar hexagonal meshing for architecture, *IEEE Transactions on Visualization and Computer Graphics*, 2015, 21(1), pp 95-106.