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Experimental study on the behaviour of masonry pavilion vaults on spreading supports

M. Rossi*, C. Calvo Barentin, T. Van Mele, P. Block

Institute of Technology in Architecture, ETH Zurich, Stefano-Franscini-Platz 1, 8093 Zurich, Switzerland

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ABSTRACT

Over the years, historic unreinforced masonry vaults have been proved to be particularly vulnerable to dynamic actions and large displacements. This paper focuses on the investigation of the structural behaviour of a pavilion vault on spreading supports by means of experimental tests on a 3D–printed scale model made of discrete blocks. Both the collapse mechanisms and the ultimate displacement capacity are analysed. The reliability of this method for investigating the collapse of vaulted structures is validated by drawing analogies with crack patterns of real vaults. The experimental data are compared with the results obtained by thrust line analysis. A good prediction is obtained by making specific assumptions that take into account three-dimensional effects, which demonstrates the importance of thrust line analysis also for 3D collapse.

1. Introduction

Historical unreinforced masonry structures have long been shown to be vulnerable to earthquakes, soil settlements, and climatic conditions, whose effects are often enhanced by bad maintenance, inadequate retrofitting interventions, change of loading conditions, etc. In the last few decades, the interest in their safety and conservation has been increasing, not only because of a growing cultural awareness of the necessity of preserving our historical heritage, but also because of their economic and social implications. Moreover, since a large portion of the world's population still resides in unreinforced masonry buildings, better knowledge of their behaviour ultimately means improved ability to save human lives.

This paper focuses on the study of unreinforced masonry vaults, which are some of the most common floor types found in both ordinary and monumental historical buildings. In particular, they suffer from the effects of dynamic actions and large displacements, and their instability is a common cause of failure, more so than the material strength [1]. Masonry is also affected by sliding. Neglecting this mode of failure may lead to an overestimation of the capacity of a structure. The complex three-dimensional behaviour of vaults, such as the one shown in Fig. 1, makes the problem even more of a challenge to understand.

In particular, this paper analyses the behaviour of pavilion vaults on spreading supports. Similar to a cross vault, the shape of a pavilion vault (also known as a cloister vault) is composed by joining four cylindrical surfaces named webs, which are determined by the orthogonal intersection of two barrel vaults. The distinction between the two types of vaults lies in the shape of the webs from which they are comprised, as shown in Fig. 2. Despite their similar geometrical origins, the two typologies show completely different structural behaviour, starting from the distribution of horizontal thrust, which is continuous along the entire perimeter of the supporting walls for the pavilion vault, but concentrated at the corner piers for cross vault.

The use of pavilion vaults has its roots in the ancient Roman imperial period. Through the development of a new construction material, the *opus caementitium*, Roman builders could experiment on wide shapes of vaults. Some of the earliest vaulted structures are found in the *Tabularium* (78–65 BCE), in the Hercules sanctuary in Tivoli (80–85 BCE), in the *Domus Aurea* (64–68 CE), in the *Domus Augustana* (81–92 CE), in the *Villa Adriana* in Tivoli (117 CE) and in several thermal baths [2,3]. After the decline of the Western Roman Empire, economic restriction leads to impoverishment of construction materials and techniques. Massive concrete vaults became too expensive and too heavy to be supported by weak masonry walls and therefore they were replaced by lighter vaults, often made of bricks or lightened with other techniques [4,5]. Although pavilion vaults were frequently used to solve the intersection between transept and central aisles of medieval churches (see, for example, the Sant'Ambrogio church in Milan, 4th–6th centuries and the San Michele church in Pavia, (9th–12th centuries), the use of cross vaults in buildings was preferred, because they did not require solid supporting walls, as their outward thrust was concentrated at the corners. During the Renaissance period (from the 16th century), pavilion vaults became some of the most frequently used floors in European palaces, exposing ceilings that were often decorated with

* Corresponding author.
E-mail address: michela.rossi@arch.ethz.ch (M. Rossi).

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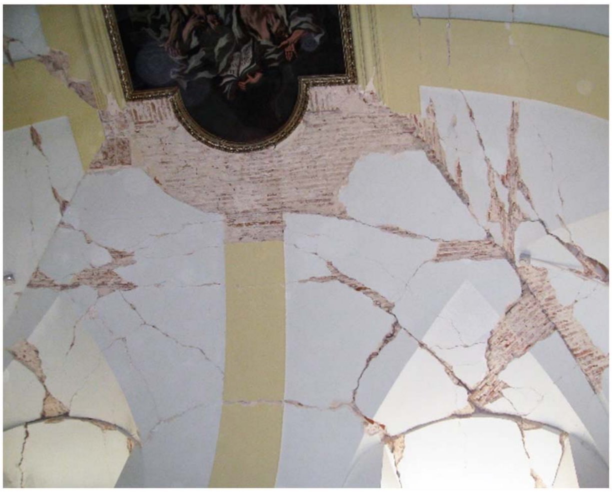


Fig. 1. Complex crack pattern of a vault in a palace damaged by an earthquake.

beautiful frescoes and stuccoes.

Since vaults plays a crucial role in the redistribution of loads to the vertical structural elements, understanding their structural behaviour is necessary not just to prevent local damages and/or collapses of the vaults themselves, but also to understand the behaviour of the whole building.

In spite of the relevance of this topic, knowledge of the structural behaviour of pavilion vaults is limited in comparison to cross vaults or domes. The "slicing technique" was one of the first approaches to investigate the state of equilibrium of vaults in which the structures were partitioned into series of arches.

Frézier [6] and Bouger [7] were the first to use this approach for the analysis of vaulted structures, specifically for cross vaults and domes. Based on these and several subsequent studies [8–10], Giuffrè [11,12] also applied the technique to pavilion vaults as an assemblage of arches. Cangi [13] proposed a mechanical model to analyse the three dimensional behaviour of pavilion vaults that originated from the slicing concept.

D'Ayala and Tomasoni [14] addressed the problem of three-dimensional behaviour by proposing an approach based on limit state analysis and showing an application to pavilion vaults. One of the crucial aspects that emerged from these studies is the presence of singularities represented by edges along the diagonals, which lead to a complex 3D state of stress, even under self- weight.

Another method for the limit state analysis of masonry vaults was proposed by [15].

This technique is based on modelling the principal stresses in a vault as a discrete force network that is constrained to lie within the masonry vault and to be in equilibrium with the applied loading pattern. Further contributions to this approach were given by [16–20]. Fraternali [21] presented a thrust network approach (TNA) for predicting the thrust surface and crack pattern of unreinforced masonry vaults, showing some numerical results for different type of vaults, among which a pavilion vault. More in-depth critical overviews of equilibrium approaches for the analysis of masonry arches and vaults are presented in [22–24].

Other authors developed non-linear continuum modelling methods suitable for 3D curved structures [25–28]. However, among all the computational modelling approaches, Discrete Element Methods have been demonstrated to be especially suitable for the analysis of masonry vaults, as shown in [29–33], in particular because of their capacity for modelling the discrete nature of masonry and the possibility of considering dynamic actions and large displacements.

In addition to these emerging computational approaches, experimental methods represent a powerful technique of analysis to simulate the studied phenomena and to provide information for calibrating and verifying the reliability of theoretical models. The use of a scale models to test the stability of masonry structures is considered a valuable method as the collapse behaviour of these structures is mainly governed by geometry rather than material strength [24,34,35]. However, some authors discussed possible effects of material properties at model scale and proposed strategies for handling them, for example using criteria of similarity and dimensional analysis [36,37].

Experiments on structures at reduced scale offer several advantages, including lower costs, easier fabrication, shorter times for preparing the setup, etc. [38]. Recent research [31,38–40] has shown the effectiveness of using the 3D–printing technique for producing small-scale models made of discrete elements to analyse vaults and domes. This technique also allows for repeated tests after damage and/or collapse caused by the application of either forces or displacements.

The aims of this paper are: 1) to analyse the characteristics of the 3D damage mechanisms of pavilion vaults caused by the spreading of supports and 2) to provide some reference values in terms of displacement capacity for future studies.

The results of a set of experimental tests on a 1:10 scale model subjected to the supports spreading are described.

In particular, the paper investigates the behaviour of pavilion vaults on spreading supports induced by the over- turning of the supporting walls, such as those illustrated in Fig. 3. This is a frequent failure mode in historic masonry building, which consists in the development of mechanisms of portions, called "macroelements", behaving like rigid blocks independent from the whole building [41–43]. Their activation mainly occurs when elements of vulnerability, such as inadequate and/or missing connections between orthogonal walls and/or between floors, are present. Fig. 4 shows three possible configurations of the wall overturning analysed in this study. Fig. 4(a) illustrates the mechanism of overturning of an external wall when its connection with the orthogonal walls fails. In Fig. 4(b), the overturning involves portions of the orthogonal walls. This can occur due to the presence of cracks in the orthogonal walls caused, for instance, by their in-plane

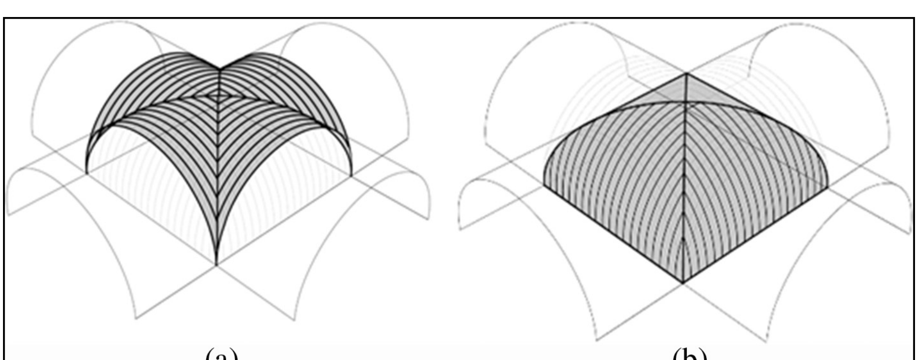


Fig. 2. Geometrical generation of (a) a cross vault and (b) a pavilion vault.

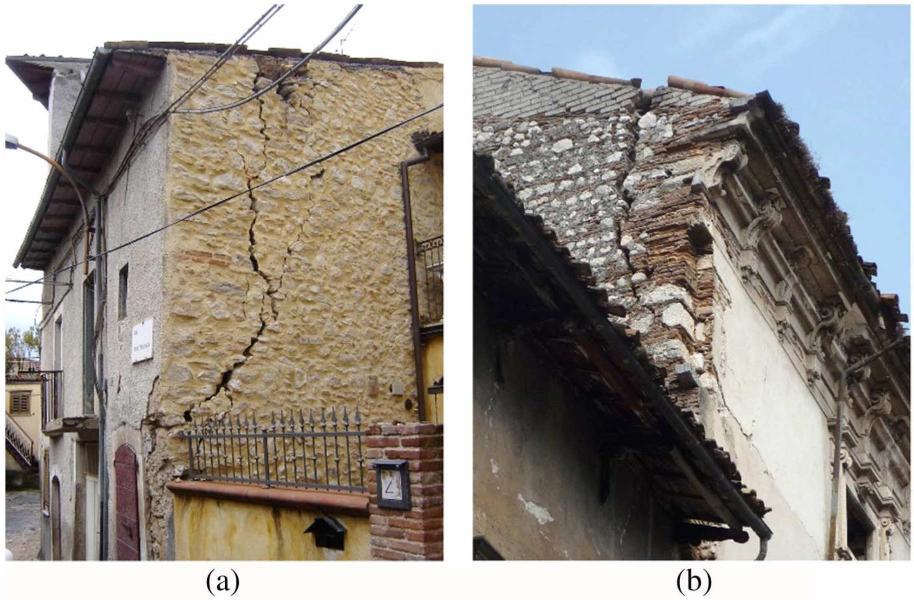


Fig. 3. Out-of-plane mechanism of (a) a whole and (b) the upper part of masonry buildings walls.

seismic response. Finally, Fig. 4(c) shows a hybrid situation. In Fig. 5, two vaults damaged by walls overturning mechanism identified in post-earthquake surveys are shown.

Given the necessity of knowing many and uncertain variables (constructive details, material properties, etc.) in order to perceive a priori what types of configurations can develop, an experimental parametric analysis is performed by varying the position of the spreading of the vault's base.

The main goals are to understand whether and how the development of the different configurations can influence the overall behaviour.

2. Experimental analysis

This section describes the experimental setup. In particular, Section 2.1 outlines the design of the pavilion vault's model (geometry, stereotomy, material, production technique), while Section 2.2 outlines the testing setup with which six different configurations of support spreading are analysed.

2.1. Scale model

The tests were performed on a 1:10 scale model made of discrete, 3D–printed blocks assembled with dry joints. All 342 blocks are printed with a ZPrinter 650 by 3D Systems with a tolerance of 0.1 mm. The material is a composite of zp150 powder and zb61 clear binder [31]. After printing, the blocks were impregnated with *Z*-bond 101 for improved strength and durability during tests. The density and friction angle of blocks are about 0.6 g/cm^3 and 38°, respectively. The geometry of the model is consistent with vaults typically observed in historical masonry buildings. A full- scale pavilion vault made of one layer of standard bricks (60 × 120 × 240 mm), with a square base of 3.5 m on each side and a rise of 1.7 m. Since usually the lower part of the vault is built merged with the supporting walls, as shown in Fig. 6, its actual springings are considered at about 30° from the horizontal plane. The resulting geometry of the scale model is shown in Fig. 7a: the internal span s is about 343 mm, the rise r 111 mm and the thickness t 12 mm. The masonry bond pattern is characterized by a shift of half a block between consecutive courses. (See Fig. 8.)

The discrete blocks of the scale model do not have the same scale proportionally as the full-scale, reference structure, since respecting the global geometrical scale would mean printing a huge amount of very small blocks (6 × 12 × 24 mm), making their assembly at the reduced scale troublesome. However, the ratio between their height and width is kept proportional to that of a standard brick, so that, as the collapse depends mainly on geometry, the crack pattern of the scale model would be proportional to that of an equivalent full-scale vault. The dimensions of most of the blocks are 12 × 12 × 48 mm except for a few that are slightly different because of practical constraints. Note that the blocks are not exactly parallelepipeds but prisms with a trapezoidal base to compensate for the absence of mortar between the joints.

Particular attention is given to build the blocks along the diagonals (Fig. 7a). Their stereotomy is carefully studied by observing historical vaults and referring to previous studies [2]. Fig. 7b shows 3D printed blocks of the vault's diagonals.

The extrados surface of each block is marked with four dotted

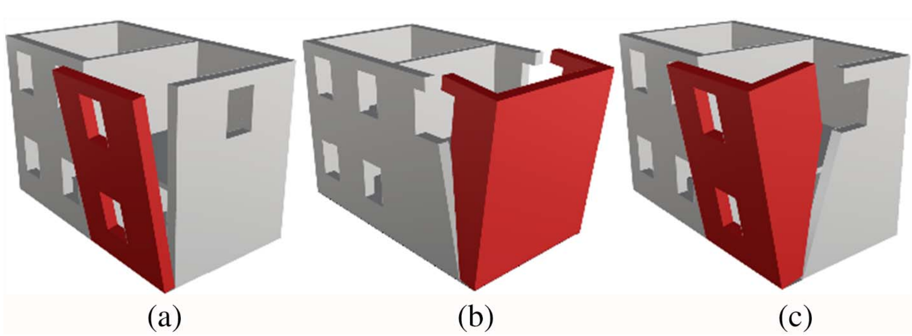


Fig. 4. Three examples of out-of-plane mechanisms of a masonry building's wall.

(a) (b)

Fig. 5. Damage mechanism of two pavilion vaults caused by walls overturning.

Fig. 6. Springing of a masonry vault in a historical palace.

targets forming a unique pattern for each block, allowing for the capture of its position and movement in space as explained in Section 2.2. Moreover, to aid the (re)assembly, each block is labelled with a number preceded by a letter (A, B, C, D) indicating the web to which it belongs.

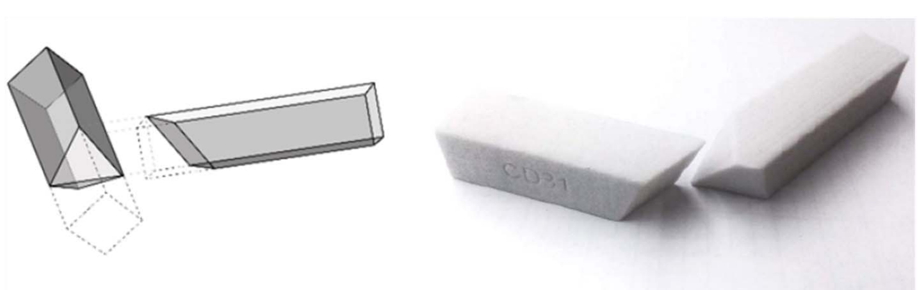
2.2. Setup

The tests were performed in the Block Research Group's laboratory at ETH Zurich using an actuated testing table, capable of applying controlled differential displacements at the base of the model. An optical measuring system allowed for capturing the development of the 3D collapse mechanisms and calculating the displacements by means of an analysis software, which can track the dotted targets on the extrados surfaces. A more detailed description of the adopted setup is presented in [17].

The vault is built with a scaffold made of intersecting cardboard frames (Fig. 9a) covered by four cardboard sheets (black in Fig. 9b). The supports are fixed wooden slats of 40 × 60 mm, which are rigidly secured to the testing table. Because of the shape of the pavilion vault, the removal of the scaffolding can be done from underneath. However, as this operation is hindered by the testing table, which does not allow full access from the bottom, the scaffolding is only lowered sufficiently to not interfere with the development of the collapse mechanisms. The testing setup is shown in Fig. 10. The spreading of the supports is obtained by the movement of three plates of the modular testing table, shown in grey, which is controlled by a linear actuator. The tests are quasi-static. The applied velocity is 0.6 mm/s. To create differential displacements, the frame is split in two parts, one fixed to the moving plates and the other anchored to the rest of the table. Since the objective of the research is to analyse the response of the vault to different possible out-of-plane mechanisms such as those shown in Fig. 4, three cuts were executed at 0, 1/4 and 1/2 of the length of the frame. As shown in Fig. 11, six cases are analysed by combining different cuts when opening the frame. In test T4, for instance, one side of the frame is left free to open in 0, while the other in 1/2.

3. Tests results

In this Section the tests results are discussed. Section 3.1 describes the static behaviour of the vaulted model before applying the outward displacement. In Section 3.2, the collapse mechanism of the vault subjected to the different spreading configurations is analysed. In each cases, the achieved ultimate displacement is reported.



(a) (b)

Fig. 7. (a) Cut of the blocks according to planes of intersection and (b) 3D printed blocks of the pavilion vault model.

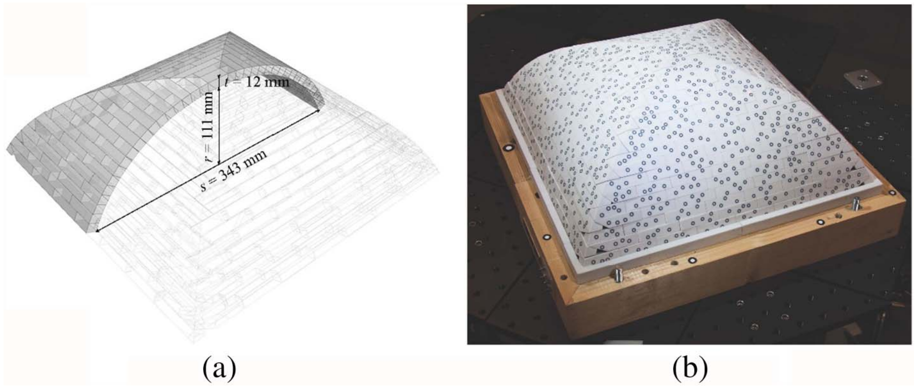


Fig. 8. (a) Dimensions of the scale model and (b) the 3D printed model of the pavilion vault.

3.1. Static settlement

Important to the understanding of the results of the dis- placement tests is the observation of the initial static behaviour of the scale model, which shows great agreement with what happens in real masonry pavilion vaults. After decentering, a slight opening of the joints appeared along the diagonals while the webs flattened. A description and interpretation of this mechanical behaviour was already given by [2], who analysed the pavilion vault behaviour by considering the webs as a series of arches, similar to flying buttresses, that extend from the diagonals to the supporting walls. Unlike the cross vault, where the intersecting webs, which carry their thrust to the convex diagonals, enhance the interlocking effect, the pavilion vault's diagonals tend to open up due to tensile forces developing in the concave diagonals. The flattening is due to outwards displacements of the supporting walls even if small that can occur because of settlement movements.

3.2. Spreading of supports

As described in Section 2.2, the three cuts of the wooden frame enable the execution of six test cases: three symmetric and three asymmetric. In each test, the following damage mechanisms occur during displacement: 1) the web orthogonal to the displacement direction (hereinafter called frontal web) is affected by a three-hinged collapse mechanism; 2) the webs parallel to the displacement direction (hereinafter called lateral webs) show the development of inclined cracks running from the position of the cut in the frame to the diagonals. Fig. 11 shows a sketch of the main damage mechanism.

Figs. 13–15 show the development of the symmetric mechanisms captured with the optical system. The three hinges, named *A*, *B* and *C* in Fig. 16, occur in different courses as a function of the cut position. The extrados hinge B forms at the eighth course in both the tests T1 and T2, while in test T3 at the course above. Hinge *A* develops along the course corresponding to the level where the lateral crack ends. With increasing displacement, the vault geometry changes and sudden jumps of hinges *A* and *C* between the voussoirs can happen immediately before the collapse, while hinge *B* always remains in its initial position.

The peculiarity of hinge *B* is that its trend is not horizontal but gradually bends towards the diagonals. This effect causes the hinge to assume an arched shape on the web's surface. This is related to the interlocking and the associated stiffness of the diagonals that obstruct the development of a clean and straight hinge line. The interlocking behaviour of the diagonals becomes less influential moving from the crown to the bottom as the proportion of the web to influence zone increases (see Fig. 12).

Figs. 17–19 show the development of the asymmetric mechanism. The failure mechanism is analogous to the symmetric one (formation of the three hinges in the frontal web and shear cracks in the lateral ones). However, hinge *A* does not follow a horizontal course, but it develops approximately along an inclined path that joins the lateral shear cracks from one side of the frontal web to the other. Hinge *B* still exhibits an arched shape, but it has the same inclination as hinge *A*. Hinge *C*, contrarily, still develops horizontally, which is most likely due to the discretisation.

Fig. 20 shows a simplified drawing of the damage pattern from top (Fig. 20a) and later points of view of both the symmetric (Fig. 20b) and asymmetric configurations (Fig. 20c). Fig. 21 shows the development of the shear crack on the lateral webs due to the sliding between the horizontal joints because of the negligible tensile strength of the material and because the friction angle between surfaces of the blocks is too small to resist the horizontal stress resulting from the applied displacement. The crack develops along an angle of approximately 30°. This value depends on the block geometry and pattern, particularly on the shape ratio of the blocks defined as $\varphi = \arctan(h/2b)$, where b is the

Fig. 9. (a) Scaffolding cardboard frames; (b) cardboard scaffolding and wooden frame on which the vault is built.

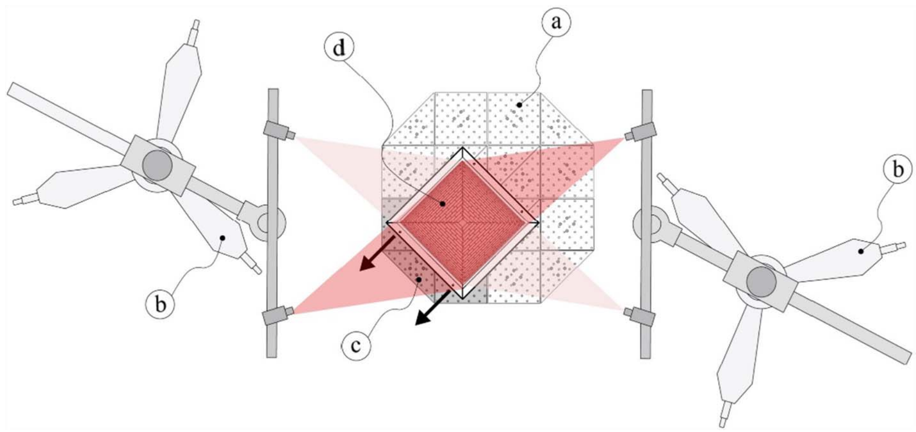


Fig. 10. Testing setup: (a) testing table with independently movable plates; (b) optical system; (c) moving plates in this experimental campaign; (d) 3D–printed pavilion vault model.

length of blocks and h is the height.

The shapes of the failure mechanisms show a significant likeness to those observed in masonry pavilion vaults, which are analysed during surveys on masonry buildings. Fig. 5 in Section 1 shows two examples of pavilion vaults in historical palaces that are damaged by the effects of walls overturning, caused by earthquakes. In particular, Fig. 5a shows a symmetric mechanism, with the occurrence of cracks along the diagonals and a clear development of the three horizontal hinges in the frontal web. Fig. 5b illustrates an asymmetric case, where the three hinges develop along an inclined line.

Table 1 shows the average values of the displacement capacity expressed in terms of both ultimate displacement d_u and its ratio to the span d_u/s. The values for the symmetric mechanisms are close to each other. The lower value is obtained in test T1, which can be attributed to the fact that there is less 3D stiffening contribution from the lateral webs on the collapse behaviour. The results of asymmetric mechanisms are even closer to each other. Moreover, these results highlight a higher vulnerability to asymmetric mechanisms compared to the symmetric ones.

The average of all the results, both symmetric and asymmetric, expressed in terms of d_u/s is of 2.63%. This value shows an affinity with those obtained by [22] that, similarly, tested a scale model of a cross vault made with discrete 3D printed blocks by applying differential displacements. In particular, the average value of du/s obtained by considering the spreading of supports was of 3.2%.

4. Thrust-line analysis

The recurring mechanism of collapse, determined experimentally, is analysed by means of thrust line analysis making use of graphic static. Even though this is primarily a two-dimensional technique, in this analysis the three-dimensional effects are taken into account making appropriate modelling choices, both geometrical and related to the distribution of loads. First, the middle arch obtained by slicing up the frontal web, hereafter called arch buttress, is considered.

The number and dimensions of voussoirs coincide with the block

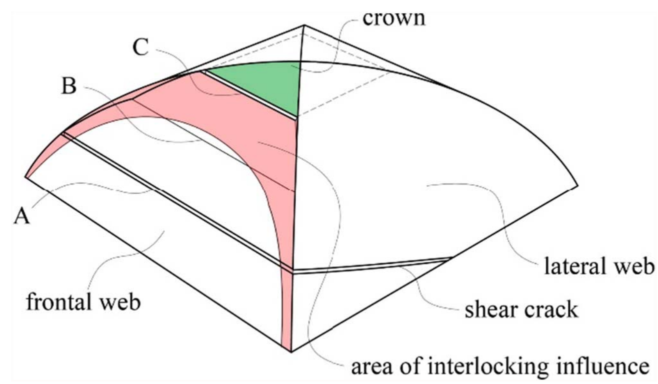


Fig. 12. Simplified interpretation of the pavilion vault behaviour. (For interpretation of the references to color in this figure, the reader is referred to the web version of this article.)

courses of the vault. The crown of the vault, which consists of the last five courses, behaves in a different way. Due to its small curvature, which is even reduced after decentering, the crown appears to be flat. Moreover, the high level of interlocking between the blocks of this region, delimited by a dotted square in Fig. 12, prevents its involvement in the three hinges mechanism. Therefore, the crown behaves like a separate stiff plate bound to the rest of the vault. In particular, as just the frontal web is analysed, the weight of only a quarter of the crown, shown in green in Fig. 12, is considered. Moving downwards from the crown to the bottom of the frontal web, the influence of the diagonals decreases. The red area in Fig. 12 shows an indicative influence area of interlocking that is delimited by an almost parabolic curve. More investigations should be done to understand the shape of this influence area.

A further crucial assumption for taking into account the three-dimensionality is the modelling of the loads acting on the arch. Instead of considering just the weight of each single voussoir, a portion of the weight of the corresponding course is added. This percentage is calculated by subtracting the portion of course affected by the inter-

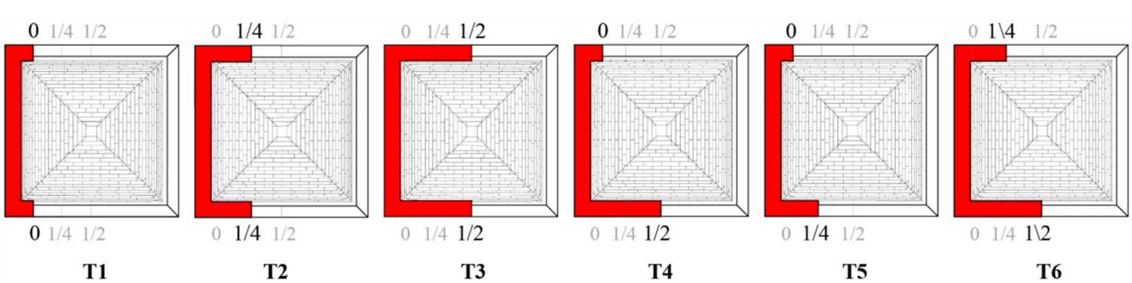


Fig. 11. Different spreading configurations analysed.

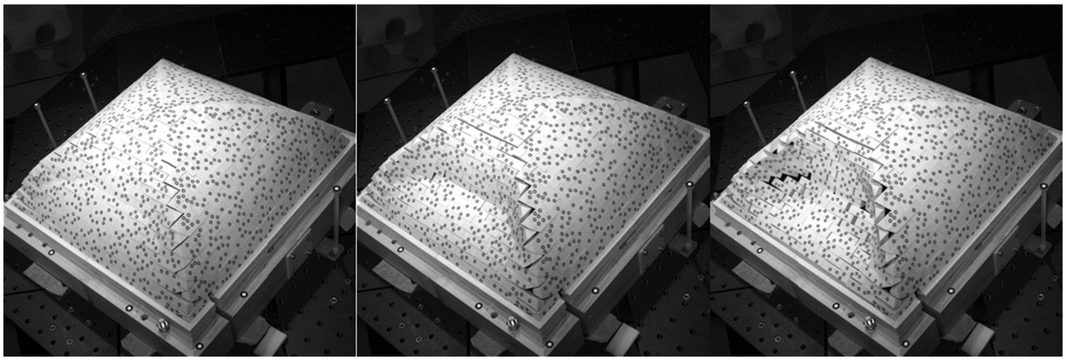


Fig. 13. Collapse mechanism of the vault's model in tests T1.

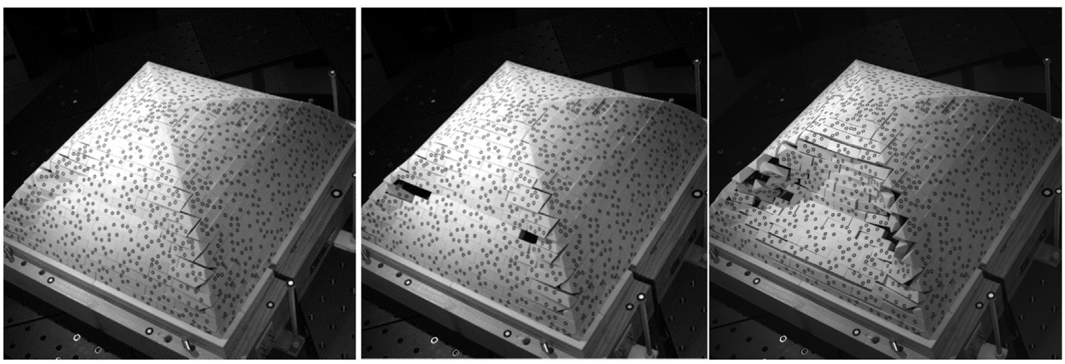


Fig. 14. Collapse mechanism of the vault's model in tests T2.

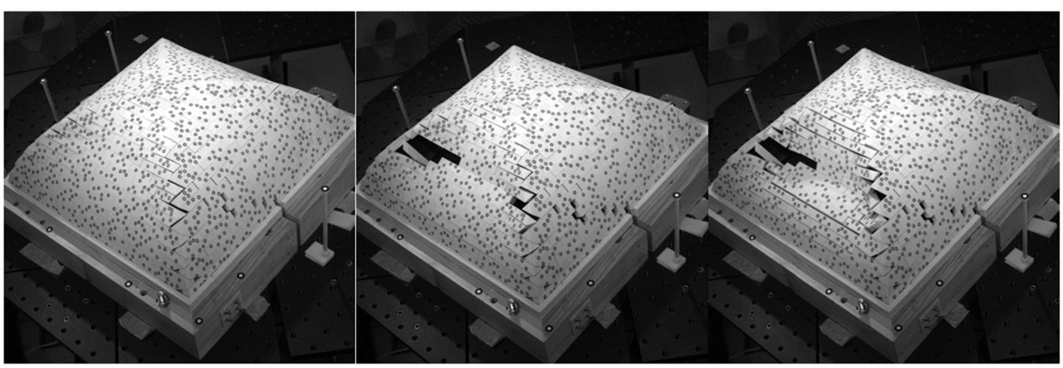


Fig. 15. Collapse mechanism of the vault's model in tests T3.

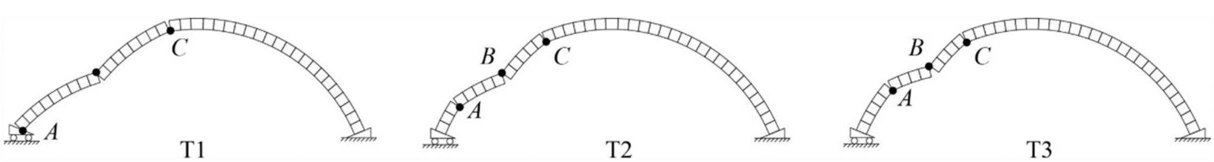


Fig. 16. Three-hinge mechanisms of the web in the three symmetric tests, where *A*, *B* and *C* are the positions of the cylindrical hinges.

locking (red area) to its total length.

Finally, the weight of the quarter crown is just modelled as an external force *P*, which depends on the horizontal thrust acting at the top of the arch.

The thrust-line analysis is carried out by using Grasshopper and Python scripting in Rhinoceros. This provides the hinge locations corresponding to minimum thrust. Three optimisation variables enter into the problem resolution: the application points of thrust at the crown and at the support, and the intersection of the start and end reaction thrusts with the resultant of the weights. Figs. 22–24 show the analysis results of tests T1, T2 and T4, respectively. The location of the hinges correspond to those obtained experimentally, except for the hinge B in T4, which is one course above that of the experiment. This can be explained by an unknown redistribution of loads due to the asymmetry.

5. Conclusions and future works

The present paper has analysed the response of historic masonry pavilion vaults on spreading supports by means of a set of experimental tests on a 3D–printed, 1:10-scale model. The influence of different spreading configurations, both symmetric and asymmetric, has been investigated. Concluding considerations are the following.

The same mechanism of failure occurs in all the tests, demonstrating

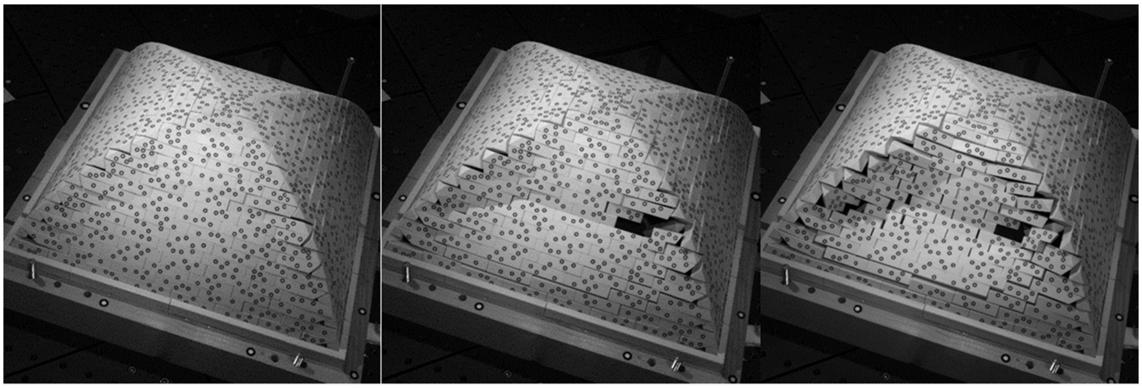


Fig. 17. Collapse mechanism of the vault's model in tests T4.

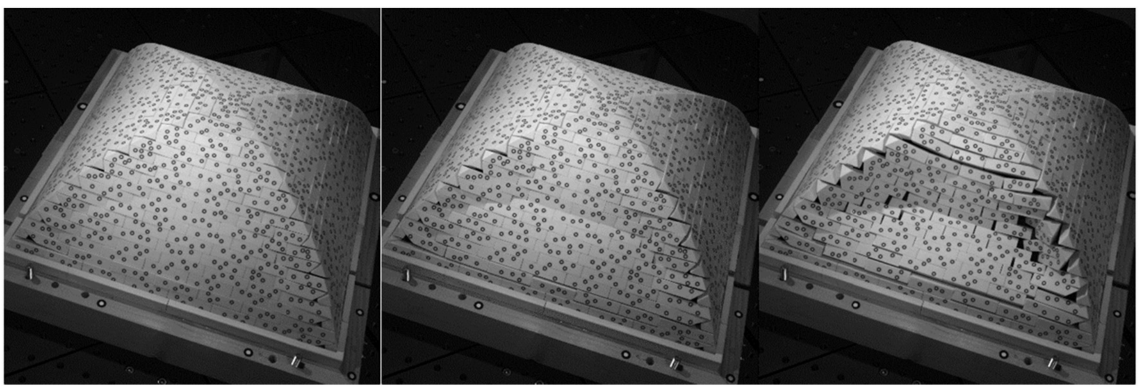


Fig. 18. Collapse mechanism of the vault's model in tests T5.

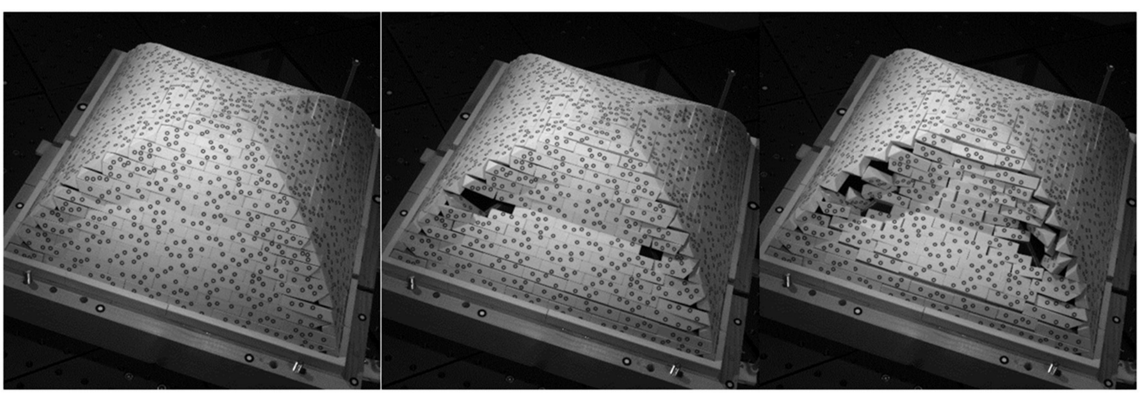


Fig. 19. Collapse mechanism of the vault's model in tests T6.

the ability of the testing setup to simulate the analysed mechanism without significant influence of the building process.

Both symmetric and asymmetric tests showed a noticeable analogy with what is observed in real masonry vaults, demonstrating the reliability of the experimental testing setup in analysing collapse.

In each test, a three-hinge mechanism occurs in the frontal web, while in the lateral webs a shear crack develops which starts from where the opening/spreading takes place;

The points on the diagonals where the shear cracks end, inform and alter the location of hinge *A* that, thus, does not always forms at the springings of the vault.

Finally, the three-hinge mechanism of collapse is graphically assessed using thrust-line analysis. The location of the hinges is in accordance with those obtained experimentally. For this purpose, specific modelling decisions are made, such as that of considering the vault's crown as an externally applied load, the definition of an area depending on the interlocking, and the resulting left-over area that informs the loads to be considered on the analysis of the arch.

This research provides an experimental benchmark for future investigation. The results of this research encourage to further investigations issues such as those listed below.

The testing setup should be improved to investigate other collapse mechanisms and loads/displacements applications. For this purpose, more research can be done using the innovative robotic technology whose adoption for testing scale model of masonry structures has been recently demonstrated at the Block Research Group's Laboratory.

Additional experimental tests on scale models is needed, varying the geometrical scale and the material properties of interfaces between blocks in order to understand their role in the behaviour of the structures.

As small-scale models are not just scaled-down simulations of real structures, but they are structures themselves, they are characterized by their own geometrical imperfections, so that also the effects of the imperfections on the structural behaviour can be investigated.

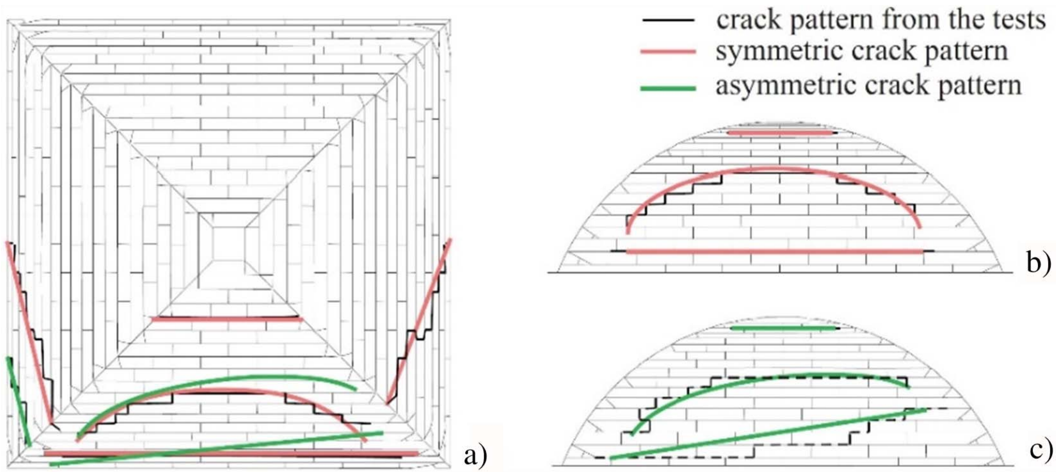


Fig. 20. Damage pattern from: (a) top view of the vault; (b) lateral view of the frontal web on the symmetric tests; (c) lateral view of the frontal web on the asymmetric tests.

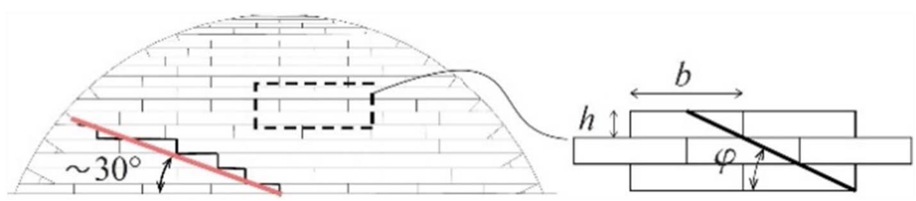


Fig. 21. Development of the lateral crack which depends on the pattern and block geometry.

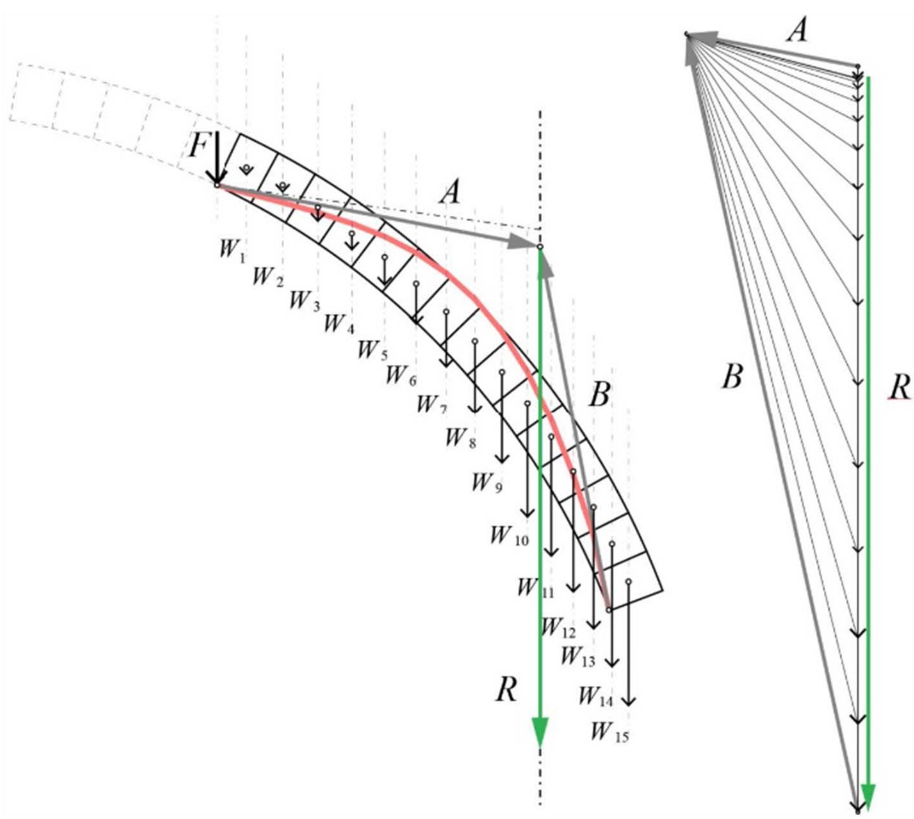


Fig. 22. Thrust-line analysis of test T1.

Additional work is demanded to not just have qualitative but also reliable quantitative data (forces and displacements capacity).

Quantitative and qualitative experimental results will be useful to calibrate discrete-element modelling [17] and thus, to make them adoptable for the analysis of masonry vaults.

Discrete element models should be used to perform sensitivity analyses varying both geometrical and mechanical parameters, in order to fully understand the behaviour of the pavilion vault under support displacements and settlements.

Further in-depth investigations are required to evaluate the role of stiffness of the webs' intersection in the collapse mechanism.

The knowledge of the failure pattern and its development should be

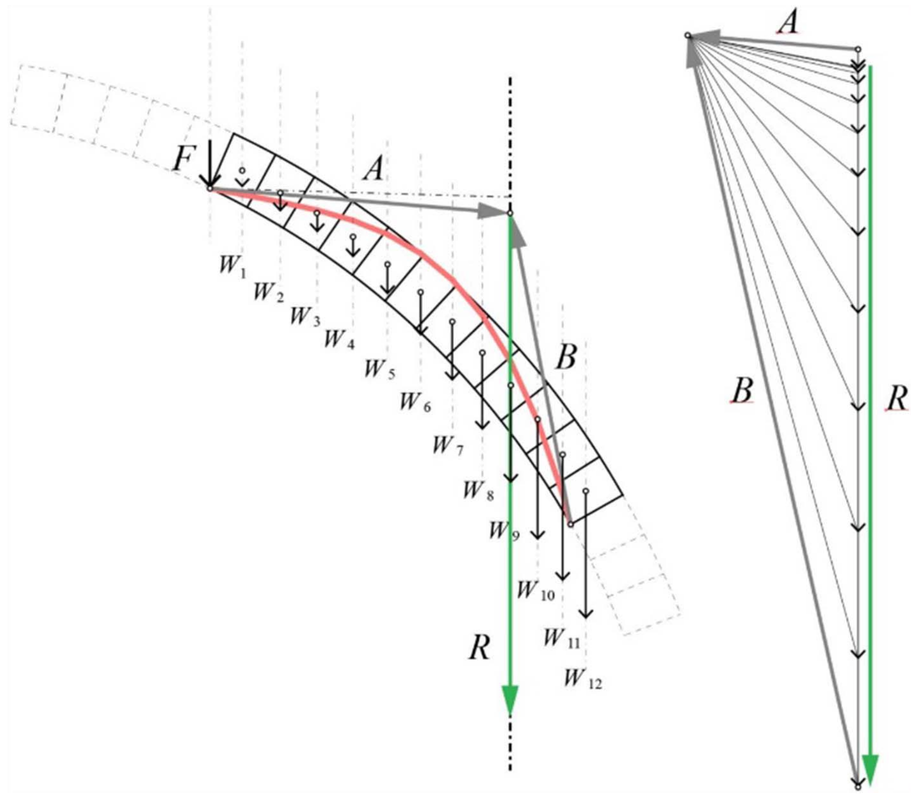


Fig. 23. Thrust-line analysis of test T2.

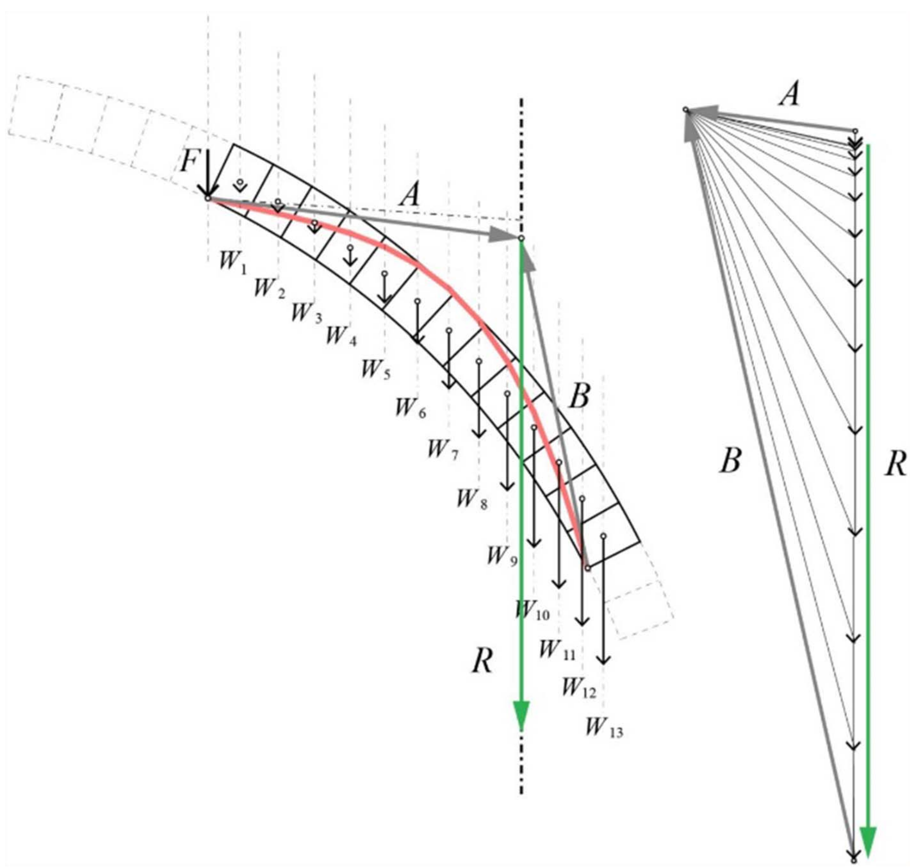


Fig. 24. Thrust-line analysis of test T4.

Table 1
Experimental results in terms of ultimate displacement and the ratio between ultimate displacement and the vault's span.

Test	d_u [mm]	d_u/s [%]
T1	9.29	2.65
T2	9.87	2.82
T3	9.51	2.72
T4	8.86	2.53
T5	8.87	2.53
T6	8.81	2.52

used to plan and design suitable retrofitting interventions avoiding the progress of cracks, which is a crucial issue, because making the structures safe before the worsening of damage means also to protect them from heavier and more invasive interventions in the future.

The effects of the fill on the structural behaviour should be investigated.

The dynamic behaviour using both experimental and numerical methods should be an interesting issue to be studied.

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