The Armadillo Vault

Computational Design and Digital Fabrication of a Freeform Stone Shell

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Abstract

This paper describes the development of an unreinforced, freeform vault consisting of 399 discrete limestone blocks with thicknesses ranging from 5 to 12 cm. The vault covers an area of 75 m² and spans more than 15 m in pure compression, without mortar between the blocks. We discuss how the design of the vault and its individual pieces was entirely driven by constraints related to the fabrication process and to the architectural and structural requirements and timeline of the project. Furthermore, we describe the form-finding process of the shell's funicular geometry, the discretisation of the thrust surface, the computational modelling and optimisation of the block geometry, and the machining process. Finally, we discuss some of the strategies that were developed for dealing with tolerances during fabrication and construction.

Keywords:

freeform, unreinforced, dry-set, cut-stone, shell, computational methods, digital fabrication





1. Introduction

Throughout history, master builders have discovered expressive forms through the constraints of economy, efficiency, and elegance, – not in spite of them. There is much to learn from their architectural and structural principles, their design and analysis methods, and their construction logics (Block et al. 2014). This paper revisits some of this lost knowledge in the context of computational geometry and digital fabrication applied to the design and construction of the presented project. It reports on the structural design, architectural geometry, and digital fabrication of an unreinforced, cut-stone vault constructed in the Corderie dell'Arsenale of the Architecture Biennale in Venice. The exhibition piece advocates for the logic of compression-only forms, not only because of their uniquely expressive aesthetics, but also because of their potential to achieve efficiency and stability through geometry.

The doubly curved vault consists of 399 individual limestone blocks or *voussoirs* assembled without mortar or other structural connections. The vault stands in compression and spans a total area of 75 m² with three linear supports along its boundary and one support in the middle. The structure has a more or less triangular shape in plan. The unsupported edges between the boundary supports create openings that provide access to the space underneath. Located in the centre of the exhibition space, the stone vault spans the central walkway between the two entrances and wraps around the existing columns. The columns penetrate the structure's surface through two large openings, one of which is partially supported (Fig. 1).

The voussoir geometry results from the discretisation pattern or *tessellation*, which determines the stone rows or *courses*. The exterior surface of the vault is called *extrados* and the interior *intrados*. The supports are made of 20 mm thick steel plates, and designed to distribute the weight of the vault as evenly as possible over the floor of the protected building. A system of steel ties connects the steel supports and absorbs the vault's horizontal thrust. The ties are necessary because no mechanical connections to the floor were allowed. Leaving them exposed shows and emphasises that the stone surface structure would not be stable without them.

Figure 2 depicts the design, analysis, and fabrication process of the cut-stone vault. The flowchart also serves as an overview of the structure of this paper. Section 2 summarises the structural and fabrication requirements for this project and defines the specific objectives of the presented applied research. Section 3 discusses the structural design of the vault, focussing on the initial form-finding process and the computational methods developed to generate the tessellation and voussoir geometry. Information about the structural analysis of the vault is not included in this paper. Section 4 describes the fabrication and assembly process. It focuses on the CAM process and the machining strategy before touching upon aspects related to the falsework and the actual assembly. Finally, Section 5 presents the completed structure and provides some concluding remarks.



Figure 2. Flowchart summarising the structurally informed, fabrication driven, computational design process of the cutstone vault. An online video documenting this process can be watched here: https://vimeo.com/167868985.

2. Structural and Fabrication Requirements

The design of the vault and its individual pieces was entirely driven by constraints related to the fabrication process and to the architectural and structural requirements and timeline of the project. In this section, we describe the structural and fabrication requirements in more detail.

2.1 Structural Requirements

First of all, the vault required an appropriate funicular overall shape that allows it to stand in compression without mortar or connections between the individual stone blocks. The form finding process of this funicular shape is briefly described in Section 3.1.

In addition, to comply with the prescribed weight limitations on the floor of the exhibition space in the protected building, the thickness of the stone shell had to be reduced to the absolute minimum. As a result, the thickness of the voussoirs at midspan of the large unsupported arches is only five centimetres, which is the minimum thickness required to avoid spalling of the stone and allow the integration of sufficiently large registration notches. High degrees of double curvature ensure that stable states of compressive stress can be developed



Figure 3. One voussoir lifted from the stone surface. Note that lifting a stone out of the structure would not be possible in reality because of the registration notches. (a) Flat extrados surface. (b) Doubly ruled, load-transferring surface. (c) Curved surface on intrados, generated with rough-cuts. (d) Registration notch. (e) Planarised, non-load-transferring side surface.

within the tight stone envelope under all loading conditions (self-weight, point loads, earthquake loads, etc.).

The total weight of the vault is approximately 23.7 tons, which is less than the load due to a crowd of people occupying the exclusion zones around the supports. To prevent too much of this weight from accumulating at the central support and also for aesthetic reasons, the overall shape of the vault is intentionally shallow. As previously discussed, the resulting outward thrust at the boundary supports is resisted by an internal system of ties. The vertical reaction forces are spread over a sufficiently large area by the footings such that the pressure underneath averages below the prescribed 600 kg/m².

Note that, due to the reduced thickness of the structure, the load-transferring surfaces between the voussoirs are small (Fig. 3). Since there is no mortar between the voussoirs, which could compensate for tolerances, these interfaces had to be flush and therefore precisely cut. This high degree of precision was (structurally) not required for the surfaces on the intrados and extrados.

Finally, the voussoirs were arranged in a staggered pattern and their load-transferring interfaces aligned to the force flow, to ensure sufficient interlocking, and to prevent sliding failure. Small male/female notches were added to these interfaces.



Figure 4. (a) 5-axis router OMAG Blade5 (Generation 3) with marked axes X, Y, Z, C and B. (b) Circular saw blades can only make planar cuts. (c) Profiling tools can create ruled surfaces.

They served primarily as registration marks during assembly (Fig. 3). Section 3.2 describes the design of the staggered voussoir pattern, and Section 3.3 the generation of the actual voussoir geometry.

2.2 Fabrication Requirements

The fabrication requirements resulted, on the one hand, from constraints of the CNC-machining process, and, on the other hand, from practical considerations regarding assembly. Due to the strict time constraints and high number of voussoirs, the main goal for the fabrication process was to limit the average cutting time. Additionally, the required precision of all bespoke stones demanded a highly accurate fabrication process.

All voussoirs for this vault were processed on a 5-axis router OMAG Blade5 (Generation 3) using a circular saw blade (Ø81 cm) and customised profiling tools (Fig. 4). For the chosen limestone, the blade allows a relatively fast cutting procedure using, for example, a maximum feed rate of 445 cm/min for a 10 cm deep cut (Rippmann et al. 2013). However, such cuts are geometrically constrained to planar surfaces (Fig. 4b). In contrast, the profiling tools can be used to process

ruled surfaces efficiently, but operate at 5 to 10 times slower feed rates (Fig. 4c). Specifically, the use of circular blades demands a convex cutting geometry along the interfaces to avoid self-intersections with the blade trajectory and thus undesired cuts in the final voussoir.

In subtractive manufacturing processes, the three-dimensional treatment of a workpiece from all sides demands the flipping of the partly processed stone block and its precise re-referencing on the machine bed. To avoid this time-consuming procedure, and potential tolerance issues, all voussoirs were designed such that their extrados is planar. Hence, they could be cut from cuboid blanks that were mounted with one planar face against the machine table. After the exposed surfaces of the workpiece have been machined, this untreated planar face equals the extrados of the processed voussoir (Heyman 1997; Clifford & McGee 2013).

An additional measure to reduce the machining time of the voussoirs was to successively cut side-by-side grooves (with a larger step size than the blade's thickness) to approximate the doubly curved intrados surface. Usually, the resulting fragile fins are first knocked off manually to then continue with finer mill passes to obtain a smooth surface finish. We decided to stop after this first step and to use the "unfinished", rough, but nonetheless precise, aesthetic as a strength by carefully aligning these grooves with the force flow (see Section 3.2).

The maximum allowed weight of the voussoirs was limited to 45 kg on the top and 135 kg close to the supports. This constraint resulted from the fact that no heavy equipment, such as mobile cranes, could be used on the construction site, and thus ensured that all pieces could be handled safely by the masons assisted only by lightweight hoists mounted on the scaffolding.

3. Structural Design and Architectural Geometry

The design of an unreinforced, discrete, dry-set, cut-stone vault with complex geometry is a complicated process. Essentially, it can be summarised by the following steps. First, a thrust surface is designed through a form-finding process. This surface is taken as the middle surface of the cross-section of the vault. The intrados and extrados are created as offsets of this middle surface according to a local thickness, defined by the live loading cases. This stone envelope is then discretised into voussoirs following a tessellation pattern taking into account the fabrication and assembly requirements. Finally, the stability of the discretised geometry under different loading conditions can be verified with discrete element modelling. In this section, we describe the design of the thrust surface (Section 3.1), the tessellation design (Section 3.2), and the generation of the voussoir geometry (Section 3.3).

3.1 Form Finding

The funicular shape of the vault is the result of a form-finding process based on thrust network analysis (Block & Ochsendorf 2007). As a first step, preliminary design alternatives were sketched using RhinoVAULT (Rippmann et al. 2012). From this, a mesh was obtained and then refined, based on functional and aesthetic considerations. The updated mesh served as a target for a "best-fit" procedure that finds the closest possible network of compressive forces under the given loads (Van Mele et al. 2014).

During the form finding process, only the self-weight of the vault was considered. As discussed in Section 2.1, the allowable self-weight was dictated by the requirements of the site and the constraints imposed on the assembly process. A corresponding thickness distribution was computed based on experience, aesthetic considerations, and common sense. As depicted in Figure 5c, the thickness varies from 12 cm at the central support and the bottom leg, to 5 cm at the highest points and at midspan of the large unsupported arches.

The layout of force directions for the horizontal thrust in the network was derived from the geometric and structural features of the three-dimensional target geometry, and represented by the form diagram in Figure 5a. The best-fit algorithm was used to find the specific distribution of forces along those directions that maps the three-dimensional network as close as possible to the geometric target. During this process, the geometric target was updated to be able to find solutions that better distribute stresses along the supports and introduce more double curvature. The force diagram in Figure 5b is the final "best-fitting" distribution. Figure 5d is a visualisation of lumped stresses at the nodes. It shows that stresses are extremely low and do not even exceed 0.1 MPa. Note that this is two orders of magnitude below the compressive strength of the selected stone, which is a Cedar Hill limestone with a compressive strength of 22 MPa.

Finally, the resulting thrust network can then be converted to a mesh that, after subdivision and smoothing, represents the middle surface of the stone envelope of the vault.

3.2 Tessellation Design

The design of the tessellation geometry is subject to a comprehensive set of constraints derived from structural and fabrication requirements (Section 2), and from aesthetic considerations regarding tectonics and rhythm. Basically, the tessellation pattern must be staggered to ensure an interlocking voussoir arrangement and properly aligned to the force flow to prevent sliding failure, particularly along the unsupported boundaries.

The design of the tessellation pattern starts with the definition of course lines on the thrust surface. The thrust surface is represented by a quad mesh (Fig. 6a), whose faces are aligned with the layout of forces defined during the form-finding process (see Section 3.1). This mesh gives a first indication regarding the orientation, singularities and spacing of the course line layout. The actual design of the courses was created manually from a set of geodesic curves on the thrust surface. Custom design and monitoring tools were used to help control the predefined minimum and maximum spacing of the courses such that, for example, the allowed weight of the average voussoir per row was not exceeded, while maintaining local alignment to the force flow (Fig. 6b).

A set of vertical lines was then generated per course aligned with the force flow. These lines define the side-by-side cuts resulting in the rough break-off edges on the intrados of the structure. A particular challenge was the alignment of cut lines from one course to the other such that the force flow becomes globally apparent (Fig. 6c). Constraints pertaining to the blade width and minimum and maximum allowable break-off widths had to be taken into account. The continuity of the cut lines was achieved by transferring the endpoints within one course to start points within the next. Given the varying geometry of the vault and the above-mentioned constraints, a strategy was developed for the gradual insertion or removal of additional cut lines.

Subsequently, an initial tessellation topology was defined by choosing more or less equally spaced vertical joint lines from the rough-cut pattern. The use of alternating boundary conditions for neighbouring courses guaranteed an initially staggered configuration. Locally, especially close to singularities, this tessellation topology was further modified manually. A more balanced staggering with larger overlaps between voussoirs was then created through an automated procedure that maximises the distance between joints of neighbouring courses. Ideally, the voussoirs of neighbouring courses are thus staggered by half of the length of one voussoir (Rippmann & Block, 2013; Rippmann 2016). In this iterative solving procedure, the vertical joint lines were automatically aligned with the local rough-cut pattern.

The final tessellation geometry was created by making all faces convex (Fig. 6d). This was achieved by scaling the vertical joint lines based on a user-defined scale factor and proportional to the course height. As a result, the degree of convexity increases towards the top, forming smoother transitions around the singularities.

3.3 Voussoir Geometry

The voussoir geometry was generated based on the tessellation of the thrust surface (Fig. 6d) and the chosen thickness distribution (Fig. 5c). The geometry of each of the surfaces of a voussoir (i.e. the intrados and extrados surfaces, the load-transferring side surfaces, and the non-load-transferring side surfaces) was determined by the limitations of the fabrication process and the limited amount of time in which the voussoirs had to be produced.

Each voussoir is convex and has a flat extrados surface. The non-load-transferring side surfaces (the surfaces transverse to the course lines) are also flat.



Figure 5. (a) The form diagram lays out the directions of horizontal forces in the three-dimensional thrust network. (b) The force diagram contains the force magnitude along each of the directions in the form diagram. (c) The distribution of thickness. (d) The stress in the surface resulting from the distribution of force and thickness.

The intrados surface of the voussoirs is curved like the intrados of the vault. It is created with parallel cuts by a circular blade leaving fins that are hammered off. The primary load-transferring surfaces (the surfaces aligned with the course lines) are ruled, because they are cut with a cylindrical profiling tool that creates the (male and female) notches.

Since the surface of the vault has areas with negative Gaussian curvature (i.e. in some areas it is anticlastic), it is not possible to create a connected flat-panel discretisation of the extrados with only convex faces (see Krieg et al. 2014; Li et al, 2015; Pottmann et al, 2015). Therefore, the extrados surface of each of the voussoirs was planarised individually creating a disconnected discretisation of the exterior of the vault.

The planarisation process is summarised in Figure 7. First, disconnected planar faces were based at the normal of the centroid of the original, smooth extrados



Figure 6. Overview of the tessellation design: (a) the mesh representing the thrust surface, (b) the course lines, (c) the rough-cut pattern on the intrados, and (d) the final tessellation of the thrust surface and aligned rough cuts.

surface (Fig. 7a). Note that this created large, erratic deviations from the original curved extrados at the corners of the voussoirs. These deviations were not aesthetically pleasing and significantly increased the weight of the vault. Therefore, in a second step, the planar faces were allowed to rotate around the normal at the centroid and move slightly up and down. The normals at the corners were allowed to rotate as well (Fig. 7b). During this procedure, the tessellation of the intrados was kept fixed. This means that the vectors connecting corresponding top and bottom corners of the voussoirs were no longer perfectly aligned with the normals of the thrust surface. However, this deviation was limited to 5 degrees from the original normal vector (Fig. 7b). Finally, in a post-processing step, the non-load-transferring faces, and the notch lines were added.

After this optimisation process, the stepping from one stone to the next was between two and five cm in all locations. The lower bound was introduced to maintain a uniform and balanced appearance. The final configuration gave the vault a slightly rough, scale-like exterior that contrasts its smoothly curving interior surface.



Figure 7. Overview of the design of the voussoir geometry: (a) disconnected, planar faces based at the centroids of the tessellation cells on the extrados create a rough, scale-like exterior with large deviations at the corners; (b) by rotating the faces and corner vectors a more balanced stepping from every voussoir to its neighbours is created; (c) deviation from allowed minimum and maximum corner stepping before normal adjustments (red voussoirs are outside of the imposed bounds); and (d) deviations afterwards.

4. Fabrication and Assembly

The fabrication and assembly of the cut-stone vault is a combination of traditional and digital methods, aimed at constructing the designed geometry with very high precision. The geometry of each voussoir is digitally processed and G-code is generated for CNC machining. The stones are cut using three different CNC machines to achieve a result that has very small tolerances and the desired finish, while keeping to a very tight schedule. The vault is assembled much in the same way as traditional masonry vaults were. Each voussoir is fully supported by a custom-made falsework. The stones are manually set, using shims, starting from all sides at the bottom and converging towards the "keystones" at the top.

4.1 CAM Process and Fabrication

The fabrication process of each voussoir starts with cutting a cuboid blank from a rough block of limestone (Fig. 8a). Its dimensions are defined by the bounding box of the voussoir. To save time and material during the cutting process of all

blanks, the 399 bounding boxes from the vault's voussoirs are categorised in 55 different sizes. The Pellegrini Single Wire Saw CNC machine is used to cut stone plates, which are then cut transversally and longitudinally according to the predefined ashlar sizes using a 3-axis blade-saw CNC machine.

The final shape of each voussoir was cut with the 5-axis CNC machine. To speed up the process, two cutting areas on opposite corners of the machining table were used, such that an already finished voussoir from one cutting area could be replaced while the next stone was being cut in the other area. Vacuum pods of different sizes and heights were used to hold the blanks in position and to keep the cutting tool from colliding with the table (Fig. 8b). Once the blank was in position, the side edges were cut with a circular saw (Ø81 cm). The result is a planar approximation of the final side edges. The next step is to shape the top face of the blank, which corresponds to the vault's intrados, by a series of side-by-side cuts (Fig. 8c). To save time during the cutting process of these grooves on top of the blank, the saw changes the direction of its trajectory each time it finishes a cut, tracing a zig-zagging path. Then, the fragile fins that result from the gap left between cuts are knocked off manually with a hammer.

To control the visual appearance of the pattern formed by the leftovers of the fins, the lead-in and lead-out of the circular blade was varied to create shallower, incomplete cuts at specific locations on the intrados (Fig. 9a). At these locations the fins would break off slightly higher (Fig. 9b), creating a balanced distribution of "highlights" on the rough, but overall smoothly curving surface.

The interfaces were finished using three different profiling tools (Fig. 8d). A simple cylindrical tool was used to finish the interfaces without registration notch. A tool with a 12 mm diameter semi-circular ridge was used to cut the female edges and one with a 12 mm diameter semi-circular groove to cut the males. To prevent potential tolerance problems, all side cuts were made following a right-to-left direction. With this cutting strategy, the tool always entered from the same side, and the rotation of the tool in relation with its trajectory was always the same as well. The resulting tolerances of the cutting process with the 5-axis CNC machine are between 0.4 and 0.8 mm.

The G-code of each voussoir was generated using dedicated CAM software after importing the cutting geometry of the voussoirs from Rhinoceros. This imported geometry contains the surfaces that define the shape of the voussoir and additional geometric elements used to define the cutting paths.

4.2 Falsework and Assembly

The voussoirs are assembled on top of a custom falsework consisting of standard scaffolding towers that support four marine-grade plywood waffle structures, one for each vault support (Fig. 10a). To minimise the amount of required material, the waffles are designed on separate orthogonal grids aligned to the main directions



Figure 8. Fabrication process: (a) The stones cut at the bounding boxes of the voussoirs. (b) Placement of the voussoir bounding box cut on the milling bed. (c) Rough cuts with a circular blade to create the intrados. (d) Processing of load-transferring side faces with custom-made tool to create the notch lines. (e) Adjacent stones test-assembled to verify alignment of rough-cut lines.



Figure 9. The break-off depth of rough-cut fins can be influenced by controlling the lead-in and lead-out of the circular blade to create a pattern of highlights on the intrados. (a) Distribution of highlights. (b) Example of different break-off depth.

of each support section. The elements in the longitudinal support directions are placed as perpendicular to the voussoirs as possible to increase stiffness. While non-orthogonal, geometrically complex waffles (Schwartzburg & Pauly 2013) could potentially be stiffer and lighter, the assembly time and complexity would be impractical and infeasible considering the tight schedule.

Each voussoir was placed on the waffle using shims, which allowed correcting the position and inclination of the voussoirs. The position of each stone was controlled with the registration notches and assessed by verifying the perfect alignment of the interfaces. In addition, total stations were used to measure the four corner points of the flat extrados of the voussoir, and compare them with the point cloud taken directly from the digital model (Fig. 10c).

Separate crews worked simultaneously on the different supports, starting from the bottom and working their way up to the "keystone" rows (Fig. 10b). This meant that imperfection and construction tolerances were accumulated at the top. The geometrical differences between the designed and the as-built vault were resolved by creating "keystones" that fit perfectly in the built geometry. The custom "keystones" were cut once all of the other voussoirs were placed and the correct shape for them had been measured on site (Fig. 10d). Note that an alternative solution is to assemble from the top down, thus taking the imperfections at the supports by grouting (Ochsendorf et al. 2016). However, considering the complexity of the support conditions caused by the loading limitations on the floor of the exhibition space, this type of corrections was not possible here.

5. Conclusions

This paper presents an overview of the structurally informed design and fabrication of the Armadillo Vault, a cut-stone vault presented at the 2016 Venice Architecture Biennale. The project was realised under extremely tight time and site constraints that drove the structural and geometrical design as well as the fabrication process.

The paper shows how a complex design and fabrication process is only possible with the aid of an integrated computational setup. The interaction and feedback from all of the steps in the process described above were an essential component to achieve such results, and to ensure a sound structure and very small construction tolerances.

The experience of the stone masons also informed the process, especially in the configuration of the keystone rows and the accumulation of imperfections in these last voussoirs. The high precision of the entire fabrication process described above minimises these errors, but does not completely eliminate them, or the need for a solution to tolerances. Therefore, manual adjustments during



Figure 10. Assembly process: (a) Plywood waffle structure on top of standard scaffolding towers. (b) Voussoirs are placed starting from the supports. (c) The position of voussoirs was assessed using total stations, measuring the corner points of the flat extrados. (d) The keystones were cut after all other stones had been placed and the required geometry to compensate for the accumulation of tolerances could be determined.

assembly by the stone masons were needed due to the accumulation of tolerances. The execution of such manual adjustments can be facilitated by simplifying the voussoir geometry. For example, by optimising the geometry of the interfaces for planarity.

The resulting structure (Fig. 11) is a demonstration of how material and fabrication constraints are not equivalent to limited design possibilities, but can be the starting point for expressive and efficient structures.

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Figure 11. Finished cut-stone vault in the Corderie dell'Arsenale of the Architecture Biennale in Venice.

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