Rule-based topology finding of patterns towards multi-objective structural design

Robin Oval
École des Ponts ParisTech

Romain Mesnil
École des Ponts ParisTech

Olivier Baverel
École des Ponts ParisTech

Proceedings of the IASS Annual Symposium 2019 - Structural Membranes 2019
Form and Force
7-10 October 2019, Barcelona, Spain
C. Lázaro, K.-U. Bletzinger, E. Oñate (eds.)

Rule-based topology finding of patterns towards multi-objective structural design

Robin OVAL[*a,b], **Romain MESNIL**[a], **Tom VAN MELE**[b], **Philippe BLOCK**[b], **Olivier BAVEREL**[a]

[*] robin.oval@enpc.fr

[a] Laboratoire Navier, UMR 8205, École des Ponts des Ponts, IFSTTAR, CNRS, UPE, Champs-sur-Marne, France
[b] ETH Zurich, Institute of Technology in Architecture, Block Research Group, Zurich, Switzerland

Abstract

Design of patterns involves meeting multiple objectives regarding architecture, engineering and construction. Form finding allows to explore the geometrical design space defined by the topology of the pattern, and to optimise a geometry to meet these objectives. This research introduces rule-based topology finding for the exploration of singularities in quad-based mesh patterns by introducing a strip grammar. This grammar allows to generate and combine multiple topologies, and therefore, multiple geometrical design spaces with qualitatively different parameters. Applying rules to create hybrid topologies based on single-objective topologies that are heuristically or algorithmically generated, allows to explore hybrid geometrical design spaces in order to meet trade-offs between multiple design objectives. This research opens a new family of rule-based topology-optimisation methods that allow to tackle multiple objectives that can relate to statics as well as fabrication, construction and other design aspects.

Keywords: conceptual design, structural design, rule-based design, surface structures, patterns, topology, singularities, topology finding, form finding, multi-objective.

1 Introduction

Discrete shell-like structures consist of the assembly of structure and cladding elements, like beams and panels for gridshells or voussoirs for vaults. Designing these patterns relates to structural, fabrication and construction efficiency, and therefore to cost and sustainability. The geometry of curved surface structures can be designed using diverse geometrical exploration algorithms to achieve feasible and affordable structures. However, the topology of the pattern controls its geometry. Indeed, the geometrical design parameters are defined by the topology, which constrains the exploration of the geometrical design space, which may not contain suitable or efficient designs. Exploring different topologies allows to break this limitation and to explore a more general topological design space, by exploring multiple parametric geometrical design spaces.

1.1 Challenges for topological design of patterns

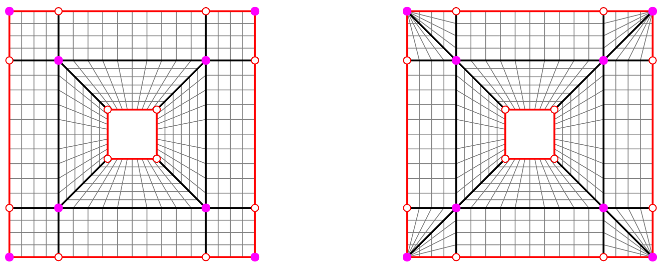
Design of patterns can face different levels of obstacles if a flexible topological design approach is not enabled. Full design opportunities are missed when the designer is forced to:

- resort to a classic quad-mesh grid relevant for very regular design problems only;
- spend time in a tedious project-specific procedure without automation process;
- algorithmically optimise the topology for a single, specific requirement; or,
- stick to a static topology all along the design process, while design objectives evolve.

Addressing these challenges would provide the opportunity to perform structural design with a dynamic topology to explore different geometrical design spaces. Such a flexibility would allow finding trade-offs between multiple, evolving, project-specific objectives regarding diverse architectural, engineering and construction aspects, like structural efficiency and fabrication affordability.

1.2 Approach to topology finding of patterns

Topological design necessitates a different approach than geometrical design. Unlike geometry, topology is not controlled by continuous-valued parameters, but topological modifications can be performed using a grammar of rules, which are designed per application. Shape grammars were introduced in Stiny and Gips [1] and evolved into functional and structural grammars (Mueller [2]) to include non-geometrical data related to structures, with multiple applications including a triangular mesh grammar for geodesic domes in Shea and Cagan [3].
Topology finding of patterns focuses on the generation and exploration of coarse quad meshes, as in Figure 1, that encode the data regarding the singularities in quad-based mesh patterns (Oval *et al.* [4]). The singularities are the mesh vertices with an irregular valency, i.e. different from four, or three on the boundary. High-valency pole points, a specific type of singularities adjacent to triangles, are integrated via pseudo-quads, which are geometrically like triangles but topologically like quads with a zero-length edge a the pole location, as in Figure 1b. The singularities constitute the core topological data, from which density and geometry are managed separately. Their exploration allows to yield a large variety of different topological design as the ones shown in Figure 2 from Oval *et al.* [5].



(a) Coarse quad mesh (b) Coarse pseudo-quad mesh

Figure 1: Coarse quad and pseudo-quad meshes, with boundaries in red, encoding the data regarding singularities, in pink, independently from density and from geometry.

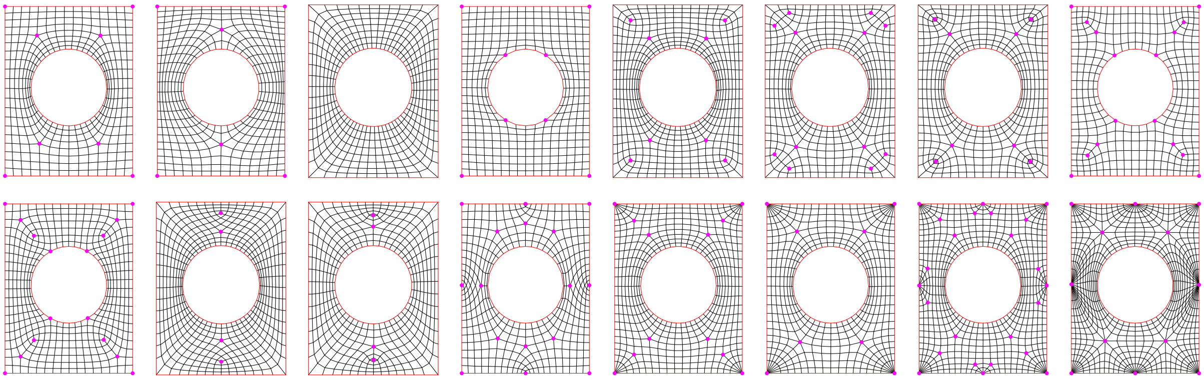


Figure 2: Topological exploration of quad-mesh patterns, with singularities highlighted in pink, for the British Museum courtyard roof from Oval *et al.* [5].

1.3 Contributions

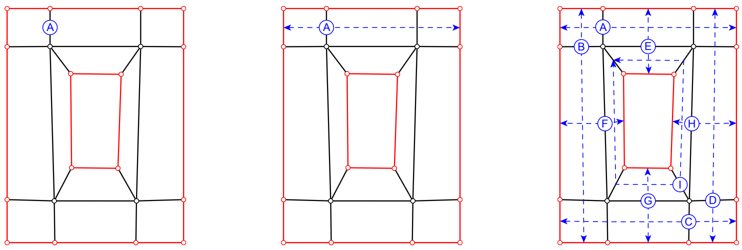
This paper introduces rule-based topology finding of quad-based mesh patterns as a means for topological exploration and structural design of patterns. The focus is set on quad meshes because they can directly describe a large variety of patterns or be converted into triangular or other polygonal meshes. This design approach enables new strategies to meet multiple objectives from statics, as well as fabrication or construction.
Section 2 develops a low-level grammar for topological exploration of coarse quad meshes based on two reciprocal rules to add and delete strips of quad faces. Generating, changing and combining topologies allows to explore multiple geometrical design spaces beyond parametric design.
Section 3 applies rule-based topology finding to the exploration of multiple topologies, aiming at finding trade-offs between multiple objectives through the generation of hybrid topologies, based on heuristically or algorithmically generated topologies for specific objectives.

2 Design grammar for the singularities in quad-mesh patterns

The design space of quad-mesh singularities is a topological space. Contrary to geometrical spaces, topological spaces do not have a metric based on continuous-valued parameters, which allow to describe and organise designs. Therefore, a rule-based approach is developed for the exploration of the singularity design space.
As opposed to a high-level grammar, as in Oval et al. [5], a low-level grammar allows comprehensive exploration of the design space with a limited number of rules. A low-level grammar consisting of adding and deleting vertices, edges or faces, for instance, can yield any type of polygonal mesh. To restrain rule-based generation to quad meshes, the low-level grammar is based on the strip structure in quad meshes, which will be introduced next.

2.1 Quad-mesh strip structure

Quad meshes have a specific structure made of strips of quad faces, which general meshes do not have, introduced in Oval *et al.* [4]. The strips in a quad mesh are collected as a list of edges by collecting edges across adjacent quad faces, as shown in Figure 3.

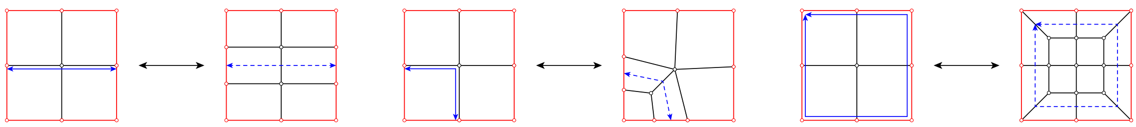


(a) Pop edge (b) Collect strip (c) Repeat

Figure 3: Collecting the face strip structure in blue in a quad mesh, with boundaries in red.

2.2 Strip addition and deletion rules

Any quad mesh can be transformed into another one with the same shape topology, i.e. the same Euler's characteristic and the same number of boundaries, by combining these fundamental strip elements. Therefore, the low-level grammar consists of two reciprocal addition and deletion rules applied to strips of quad faces (Oval *et al.* [4]). The addition rule inserts a strip along a polyedge and the deletion rule collapses a strip into a polyedge, as shown in Figure 4 for different polyedge and strip inputs. Polyedges and strips can have repeated elements, allowing overlapping and crossing. Pole points are included by converting the quad face extremity of a strip into a pseudo-quad. Combining addition and deletion rules allows to perform topological exploration of the singularity design space, as illustrated in Figure 5.



(a) (b) (c)

Figure 4: Reciprocal addition and deletion rules of strips, shown as dashed blue polylines with their corresponding polyedges as blue continuous polylines.

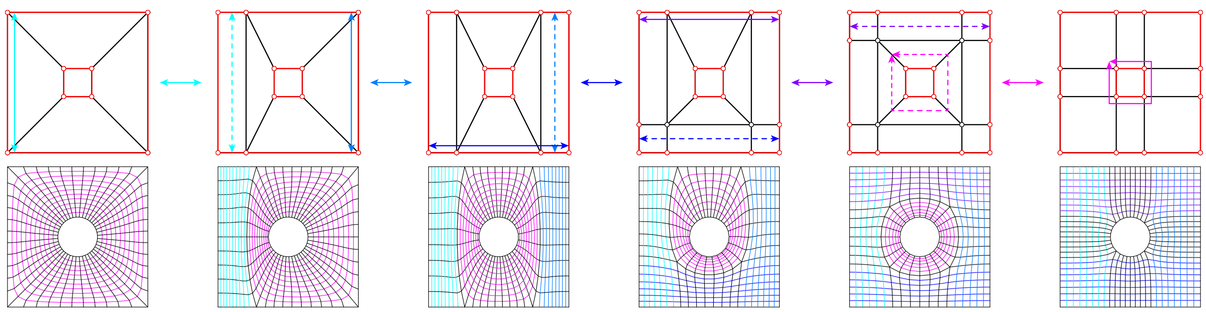


Figure 5: Combining strip rules. Reciprocal addition and deletion rules are represented in the same colour with polyedges as continuous polylines and strips as dashed polylines.

2.3 Combining topologies and their parametric design spaces

The introduced low-level grammar of strip rules allows to explore qualitatively different geometrical design spaces, whose parameters are defined by the topology. This approach can be applied to any geometrical approach, such as funicular form finding using Thrust Network Analysis (Block and Ochsendorf [6]). In Figure 6, funicular – or compression-only – shapes are explored by creasing smooth shapes along their respective pattern, concentrating the initially quasi-constant thrust distribution along specific strip polyedges. Topology A allows to add orthogonal creases to the smooth funicular shape. Multiple creases can be added but only of the same type, as constrained by the topology. Adding new strips allows to form new types of creases, as in topologies B and C with O- and U-shaped creases, respectively. The richness for topological exploration is as great as the number of combinations of strips to add. The strip rules can be enumerated separately and combined, as topology D is combining the rules that yield topologies B and C, allowing to form both O-shaped and U-shaped creases in the funicular shape. Enhancing geometrical exploration with topological exploration enables a design approach beyond only form finding.

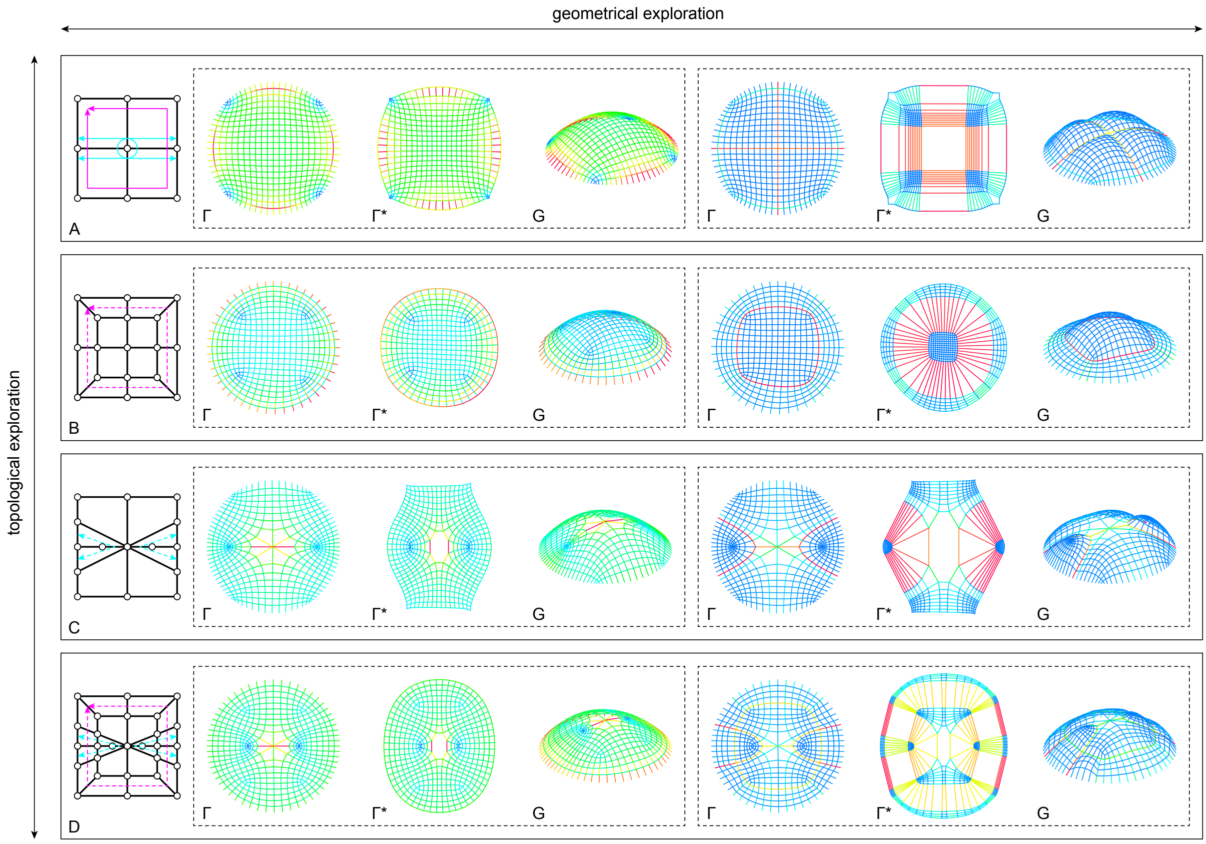


Figure 6: Exploring multiple geometrical design spaces via topological exploration of coarse quad meshes by enumerating and combining strip addition rules. The funicular shapes G derive from the form and force diagrams Γ and Γ*.

3 Application for multi-objective structural design

Rule-based topology finding of quad-mesh patterns using the presented grammar provides an exploration means to search topologies to find trade-offs between multiple requirements, starting from heuristically or algorithmically generated topologies. Hybrid topologies can be generated through the application of rules that combine the strip of the input topologies.
This concept is applied to mapping gridshell quad structures on a pillow-like shape, with a square boundary and pinned at its four corners only. The span is 10m for a rise of 2 m. The clamped, steel S235 beams have RO 114.3/4 tubular cross sections. The free-edge cross section are made around a 100 times stiffer for local stability using RO 457.2/6.3 cross sections [7].
The coarse quad meshes are densified based on a target number of faces equal to 500, constraining the strips to have the same density parameter. Surface mapping and relaxation is performed with Laplacian smoothing.

3.1 Combining objective-oriented heuristic topologies

In Figure 7, the three topologies A, B and C are generated based on heuristic rules to meet different design objectives: topology A provides geometrical regularity to ease fabrication; topology B, with four poles, provides a high number of paths towards the four corners; and, topology C, with one pole, provides a high number of paths from the surface's centre. These poles stem from interpretation of statics considerations, here the location of supports or loads. The strips of each of the three topologies are combined using addition rules to generate the four new hybrid topologies AB, AC, BC and ABC.

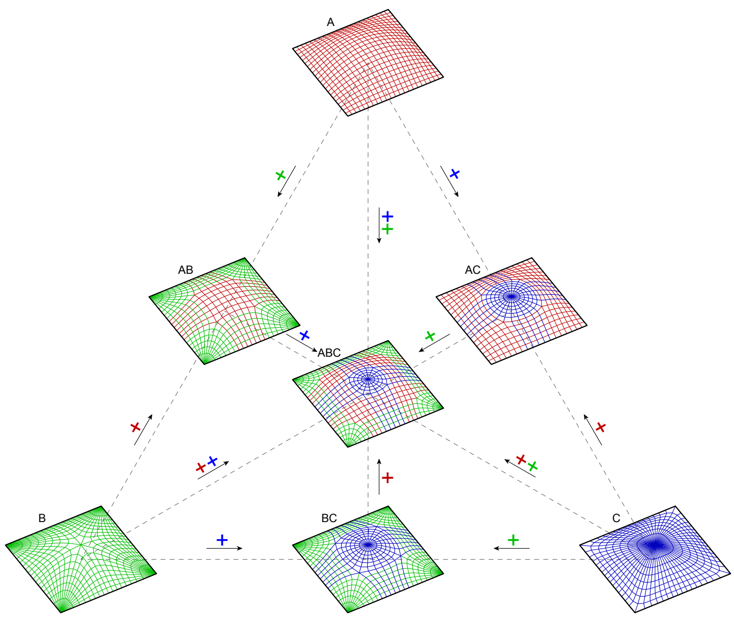


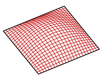
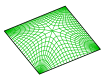
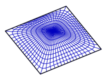
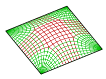
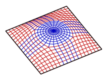
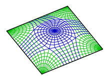
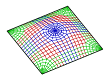
Figure 7: Rule-based topological combination of three heuristic topologies A, B, C into hybrid topologies with different strip structures. Common strips are highlighted using the same colour.

The seven designs are assessed according to three performance metrics to be minimised: the strain energy for a uniform downward surface load E_{srf}; the strain energy for a central point load E_{pt}; and, a measure of edge length disparity L, in Equation 1. The first two metrics relate to structural efficiency, using a FEA linear elastic model, and the third one relates to fabrication ease.

$$L = (l_{max} - l_{min})/l_{total}. \tag{1}$$

The relative performances, normalised by the maximum value, are displayed along with the rank in Table 1. The lower the value or the rank, the more efficient the design. The numerical results confirm the relevance of the proposed heuristics for this design problem: adding poles at the support corners induce generally stiffer structures; adding a pole at the point load location induce stiffer structures against this specific load case; however adding poles induce strong distortions in the edge lengths. None of the designs is optimal for all the performance metrics, and each features a different trade-off. It belongs to the designer to select the most relevant one regarding the project specificity.

Table 1: Performance metrics to minimise, with ranks between parentheses.

							
$\overline{E_{srf}}$ [-]	0.77 (6)	0.44 (3)	0.56 (5)	0.43 (2)	1 (7)	0.41 (1)	0.51 (4)
$\overline{E_{pt}}$ [-]	1 (7)	0.63 (4)	0.29 (1)	0.95 (6)	0.66 (5)	0.32 (2)	0.57 (3)
$\overline{L}$ [-]	0.07 (1)	0.96 (6)	1.00 (7)	0.77 (5)	0.40 (2)	0.79 (4)	0.61 (3)

3.2 Interpolating objective-informed topologies

In Figure 8, instead of being heuristically designed, the initial topologies are generated informed by the principal stress lines at the medial surface. Two different downwards surface loading are considered: a symmetrical one applied all over the surface and an asymmetrical one applied on the right half only. Two patterns are generated with the singularities of the two cross fields. Then, strip grammar rules are applied to go from one topology to the other, interpolating hybrid topologies that act as a rule-based topological gradient. Due to its combinatorial nature, for n rules to apply between two – or more – topologies, 2^n combinations of rules exist to generate hybrid topologies with different strip structures. Here, only 12 of the $2^7 = 128$ are displayed, at different rule-based topological distances or similarities between the input topologies.
Nervi-like ribbed structures based on principal stress lines [8] can be revisited, enhanced by topological exploration of hybrid pattern topologies between multiple loading conditions. Structural performance optimisation necessitates to take into account the density and geometry design spaces as well, combining both topology finding and optimisation.

Conclusion

Rule-based topology finding has been introduced for topological exploration of the singularities in quad-based patterns to explore multiple geometrical design spaces in order to meet trade-offs between multiple objectives for the design of structures. The topological design space is explored using a low-level grammar that applies addition and deletion of strips of quad faces, which modifies the singularities and poles in quad meshes. Topological exploration using these rules allows

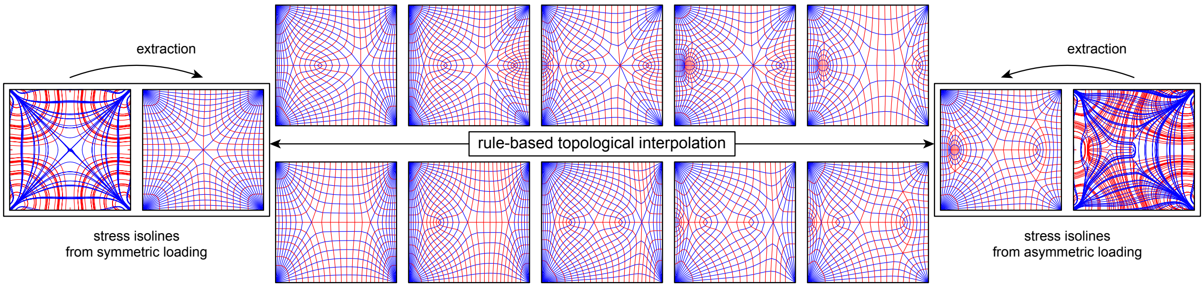


Figure 8: Interpolating hybrid topologies in a topological gradient between topologies derived from principal stress lines for different load cases.

to generate and combine different parametric design spaces for the purpose of geometrical exploration and optimisation. Yielding hybrid topologies and geometrical design spaces, based on heuristically or algorithmically generated topologies meant to meet specific objectives, allows the designer to find trade-offs between multiple objectives. This approach is complementary with existing design and analysis algorithms that are embedded in parametric design environments, providing form finding and performance evaluation for instance.
The challenges for topological design are answered: any topology, including advanced ones, can be generated; their generation is instantaneous, thanks to the direct application of topological rules; multiple objectives of different natures can be addressed, thanks to its complementarity; and, the topology can evolve during the design process. This approach opens to a new opportunity to perform rule-based topology optimisation, including multiple objectives, which can relate to statics as well as fabrication, construction and other design aspects.

References

[1] G. Stiny and J. Gips, "Shape grammars and the generative specification of painting and sculpture", in *Proceedings of the CIFIP 1971*, 1971.

[2] C. T. Mueller, *Computational exploration of the structural design space*, PhD thesis, Massachusetts Institute of Technology, 2014.

[3] K. Shea and J. Cagan, "Innovative dome design: applying geodesic patterns with shape annealing", *AI EDAM*, 11(5):379–394, 1997.

[4] R. Oval, M. Rippmann, R. Mesnil, T. Van Mele, O. Baverel and P. Block, "Feature-based topology finding of patterns for shell structures", *AIC*, 103:185–201, 2019.

[5] R. Oval, M. Rippmann, R. Mesnil, T. Van Mele, O. Baverel and P. Block, "Topology finding of structural patterns", in *AAG 2018*, Klein Publishing GmbH (Ltd.), 2018.

[6] P. Block and J. Ochsendorf, "Thrust Network Analysis: a new methodology for three-dimensional equilibrium", *Journal of the IASS*, 48(3):167–173, 2007.

[7] F. Venuti and L. Bruno, "Influence of in-plane and out-of-plane stiffness on the stability of free-edge gridshells: A parametric analysis", *Thin-Walled Structures*, 131:755–68.

[8] A. B. Halpern, D. P. Billington and S. Adriaenssens, "The ribbed floor slab systems of Pier Luigi Nervi", *Journal of the IASS*, 54(3):127–136, 2013.