Architectural and Structural Investigation of Complex Grid Systems

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Extended Abstract

Grid systems have been used extensively in the built environment, predominantly as space-enclosing shell-type structures or true three-dimensional spatial structures. They can offer a number of architectural advantages, most importantly providing and controlling lighting as well as visual relationships by the targeted composition of the structural elements. In addition, spatial grid systems offer great potential in terms of structural efficiency if an arrangement of members is chosen that permits loads to be transmitted predominantly by axial forces. However, this requirement typically leads to kinematically determinate, most often triangulated systems, severely limiting the design vocabulary available to architects and engineers. The implementation of non-triangulated patterns may offer significant advantages: greater transparency, less density (both visually and physically) and enhanced design freedom (from very regular and homogeneous tiling patterns to random, essentially free-form arrangements).

This paper describes investigations into the architectural and structural performance of complex, non-triangulated grid structures. Such structures are often highly differentiated, thus an integrative, parametric design approach in conjunction with digital fabrication technology is necessary for their realization. In addition to the considerations of geometric requirements, the parametric approach has been extended to take into account material properties as well as fabrication constraints. Furthermore, custom plug-ins to assess performance criteria of the system (static behavior, sunlight simulation) have been implemented, with the ultimate goal of providing seamless integration of 3D design software with high-level performance analysis.

In addition to the large degree of differentiation, non-triangulated systems are most often kinematic, requiring additional stabilization. One novel method is the stabilization of a two-dimensional grid by enclosing the system between two vacuumized layers of film. An investigation is described that was performed to evaluate the potential of vacuum stabilization and the significant complexities of its application to a doubly-curved structure.

To illustrate the above concepts, two case studies of recently realized full-scale grid structures of high complexity and architectural sophistication are presented in the paper. These two structures exemplify the consistent application of a digital chain from design to production as well the integrative approach to include performance criteria into the design process.
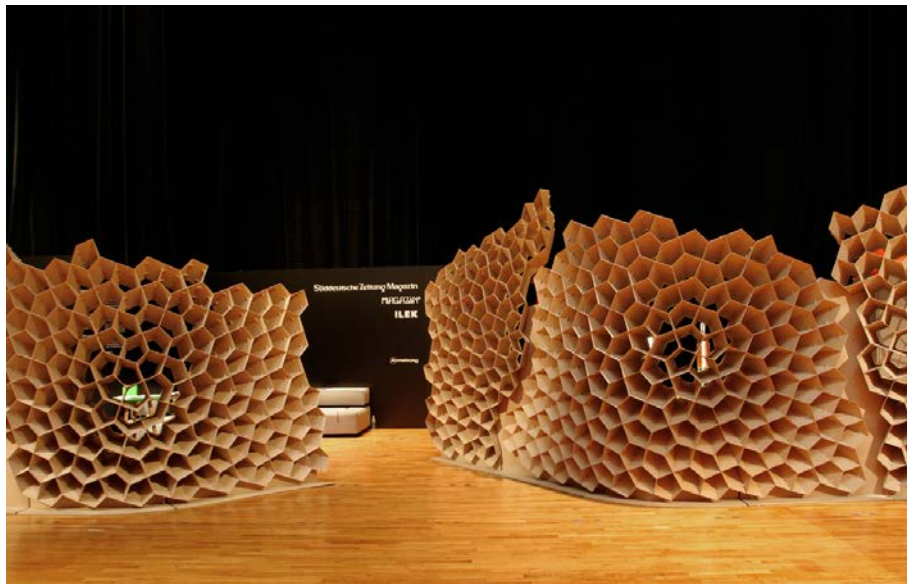

Figure 1: Case Study: 3D2REAL

Figure 2: Case Study: rn601

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Abstract

This paper describes investigations into the architectural and structural performance of complex, non-triangulated grid structures. Complex grid structures are often highly differentiated (i.e. having many components of different geometry), thus an integrative, parametric design approach is necessary. In addition to the considerations of geometric parameters, the evaluation of architectural and structural performance criteria, including automated interfaces, as well as tools to assess the stability of such systems as well as sunlight simulation are described.

Non-triangulated grid systems are often kinematic and require stabilization measures. A novel method of using vacuumized film to stabilize a doubly curved shell structure with a non-stable grid arrangement was investigated.

Two case studies of full-scale grid structures of high complexity are presented in the context of parametric design, digital fabrication, and integrative design processes.

Keywords: Grid Structures, Parametric Design, Digital Fabrication, Static Determinacy, Vacuumatic Stabilization

1 Introduction

Grid systems have been used extensively in the built environment, predominantly as space-enclosing shell-type structures or true three-dimensional spatial structures. Their inherent partial transparency can serve a number of architectural purposes, most importantly providing and controlling lighting as well as visual relationships by the targeted composition of the structural elements. In addition, spatial grid systems offer great potential in terms of structural efficiency if an arrangement of members is chosen that permits loads to be transmitted predominantly by axial forces, minimizing bending. However, this requirement typically leads to kinematically determinate, most often triangulated systems, severely limiting the design vocabulary available to architects and engineers. The implementation of non-triangulated patterns may offer significant advantages: greater transparency, less density (both visually and physically) and

enhanced design freedom (from very regular and homogeneous tiling patterns to random, essentially free-form arrangements).

2 Basic Structural Principles

2.1 Static and Kinematic Determinacy

Grid systems are most commonly composed of individual, typically straight elements. To provide a continuous, stable load path, the arrangement of elements and the connections between them are of critical importance. The most efficient systems from a structural point of view are those that allow members to carry mostly axial forces. This enables a uniform stress distribution and thus a better use of the section (fully-stressed-design approach) as well as moment-free joints. However, such pin-jointed assemblies require specific arrangement of members in order to achieve structural stability (either statically determinate or indeterminate). Most often, this leads to triangulated arrangements of members. It can easily be seen that a triangular system is stable, while a quadrangular is not (ref. Figure 1).
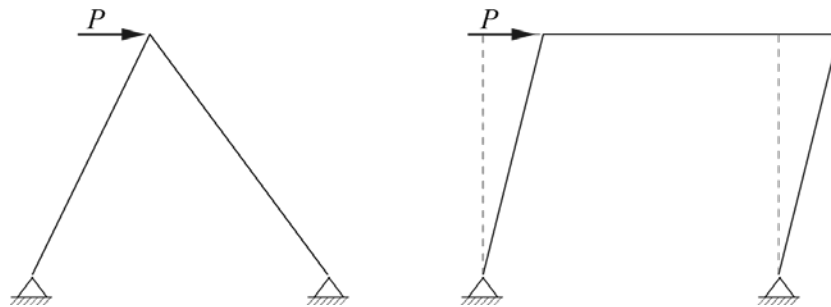

Figure 1: a) Stable triangular and b) unstable quadrangular system

Several methods are available to evaluate the stability of pin-jointed assemblies. A simple way is a formula known as Maxwell's rule ([1], [2]). It requires for static determinacy in 2D that

$$b = 2 \cdot J \qquad (1)$$

where b is the number of bars, and J is the number of free (non-foundation) joints. However, Maxwell's rule is a necessary, but not a sufficient condition for static determinacy. The arrangement in Figure 2 for example satisfies Maxwell's rule with $b = 8$ and $J = 4$, but it can clearly be seen that one portion of the structure is kinematically indeterminate (a mechanism), while another portion is statically indeterminate (redundant).
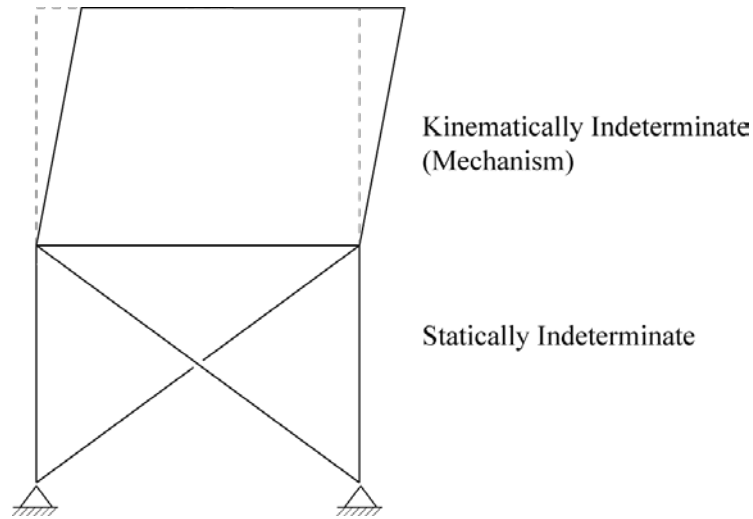
Figure 2: Non-stable system fulfilling Maxwell's rule

A more thorough method is based on matrix structural analysis and uses the equilibrium matrix to assess static determinacy ([3], [4]). The equilibrium matrix is derived from the equilibrium equations for each free joint. In 2D (ref. Figure 3) the equilibrium equations at a joint can be stated as

$$P_{ix} = Q_{k,x} + Q_{l,x}$$
$$P_{iy} = Q_{k,y} + Q_{l,y}$$

(2)
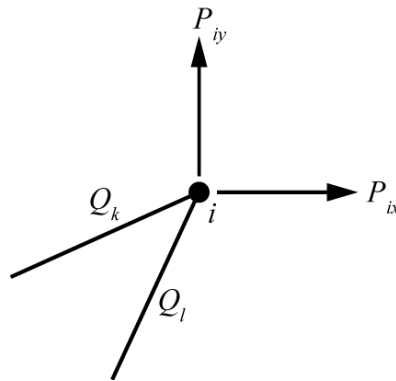
Figure 3: 2D Equilibrium at free joint

Expressing the equilibrium equation for all free joints in matrix format yields:

$$[P] = [B] \cdot [Q]$$

(3)

where [*P*] is the vector of applied forces (at free nodes), [B] is the equilibrium matrix, and [*Q*] is the vector of member axial forces. The relationships shown for 2D in Equations (2) and (3) are easily expanded to 3D by including the equations of equilibrium in the z-direction. Thus, the equilibrium matrix is of size [*m* x *n*] with *m* = number of free degrees of freedom (equal to number of elements in [*P*]) and *n* = number of member forces (equal to number of elements in [*Q*]).

Determining the rank of [*B*] will yield conclusions about the static and kinematic determinacy of the system: if the rank of [*B*] < *m* then the structure is kinematically indeterminate (unstable mechanism); if the rank of [*B*] < *n* then the structure is statically indeterminate (i.e. the system of static equilibrium equations alone is insufficient for a force analysis). For the system shown in Figure 2, the rank of [*B*] is equal to 7, while *m* = 8 (number of free degrees of freedom = number of joint forces) and *n* = 8 (number of member forces), thus the structure is in parts both statically and kinematically indeterminate.

Assembling and determining the rank of the equilibrium matrix for any given structure by hand is tedious. To facilitate this analysis for any 2D or 3D system, a custom plug-in was written in Rhinoscript® that allows outputting the relevant parameters (joint coordinates and member connectivity) for any geometry created graphically in Rhinoceros®. Additionally, Matlab code was written to read in these data, automatically set up the equilibrium matrix, and evaluate the static and kinematic determinacy of the structure

Using these efficient tools, a first assessment can quickly be made regarding the structural behavior of the system and stabilization methods required, prior to a more thorough analysis using for example FEA software. The seamless integration of custom plug-ins such as those described, as well as more sophisticated analysis tools during the design process is essential for the realization of projects with complex geometries.

2.2   Concepts of stabilization

While triangulated systems are generally stable (i.e. kinematically determinate) they provide little architectural freedom. If a non-triangulated geometry is to be used, measures must be taken to prevent shear deformation of the individual panels (ref. Figure 1a)). Such measures may be

   a. Cross-bracing, either tension-only, or tension-compression, likely resulting again in triangulation.

   b. Moment connections, preventing rotation between the members.

   c. Surface elements to resist shear deformation (similar to shear walls), such as stiff panels or foil elements.

   d. 3D bracing schemes of higher complexity, such as was employed for the Eden Project [5]

e. Using particular member arrangements, that under certain conditions (in particular with pretension) are only first-order mechanisms, but provide resistance to large displacements (for example, tensegrity-structures, ref. [6])

3 Architectural Considerations

A thorough understanding of the static and kinematic determinacy of grid systems is crucial when architectural and visual requirements correlate with the structural form and pattern of the grid. Usually the outer and inner appearance of light and transparent structures is directly linked to the visible structural layout. Therefore it is essential to find ways to change structural and architectural parameters of 3D grid systems in a reciprocal manner. For grid systems this inter-relationship of function, structure and appearance can be further investigated by considering issues of lighting and visual relationships, the structural behavior and construction as well as the emergence of ornaments and patterns.

The control of lighting and visual relationships is an essential element for architects to filter and define space in nearly every built structure. Moreover natural sunlight is used to provide light and thermal energy in winter but usually has to be screened or filtered in summer. To simulate the structure's capability to block sunlight and certain views from different directions, a ray tracing algorithm can be used. The data provided by this procedure can then digitally inform a 3D model linking functional, structural and visual information at the same time. It is possible to change the different input parameters of the system and visualize the resulting geometry. For example the planar structural elements and the individual cells of a 3D grid system as shown in Figure 4 can vary in size and orientation to fulfill certain criteria by means of rule-based models.

Architectural expression is a function of visible structural layouts and tessellations of grid systems and shell structures. Therefore the system-inherent ornamentation evolves from the alignment of the structural elements. The implementation of structural ornaments in architecture dates back to very early built structures. But whereas traditional ornaments played an important role in architecture until the 19th century entirely new ways of ornamentation started to arise with the beginning of the information age [7]. Computer-aided methods of design and production led to highly differentiated façades and structural framework with numerous individually designed and fabricated elements.

The example in Figure 5 shows the parametric transformation of a pattern known as the Cairo Tiling. By changing certain input parameters the pattern can vary locally. This principle can be used for the design of grid systems where elements change length and orientation due to functional, visual and structural requirements.
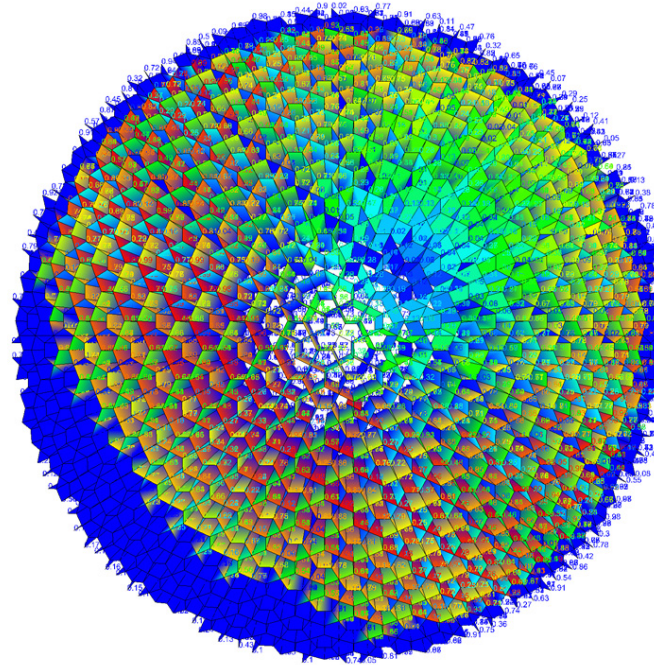
Figure 4: Sunlight simulation (using a custom programmed plug-in) of a grid shell structure with deep planar elements; visualization of the numeric data
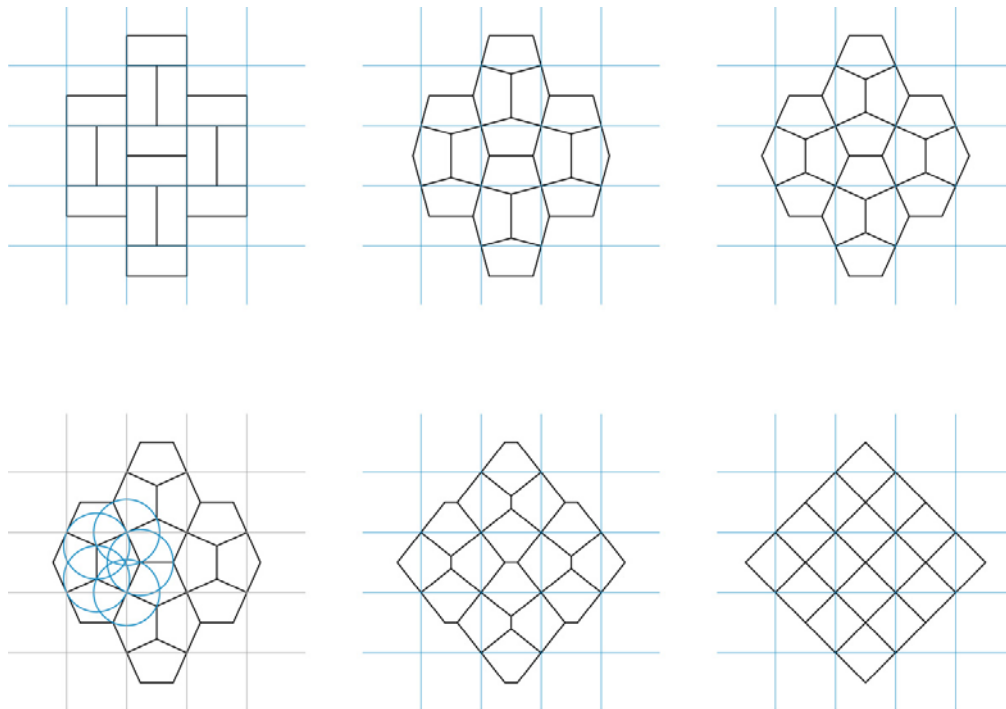

Figure 5: Parametrically transformed Cairo pentagonal tiling

4 Digital Design and Production Methods

The design and construction of complex 3D grid systems as described in the previous chapter require new processes of planning and fabrication. Within the last few years, the advent of digital and parametric tools led to various built examples of such structures.

In this context the digital model serves no longer only as a visual design environment for architects and engineers. The reciprocity and complexity of the main parameters within the overall design process leads to digitally informed models containing data that is not only generated but also processed, stored and exchanged in a bidirectional manner. The digital chain starts with the design informed by concepts, simulations and analysis and continues with optimization procedures to generate the relevant data for digital production processes.

Irregular grid systems have a large variability of components. Digital tools like CAD and FEA programs are used to process data of such non-standard structures. Due to the systems' complexity and irregularity customized scripts and plug-ins are used to extend the programs capabilities. Building upon this setup digital fabrication methods are used to produce the individual elements of the structure. Although (at least for the time being) the digital chain typically ends at the fabrication process, evolving constraints from production techniques and material specifications are integrated as input parameters from the outset.

5 Case Studies

Two recent projects at the Institute for Lightweight Structures and Conceptual Design have been planned and realized to combine structural and architectural aspects as outlined above. More importantly, they specifically took advantage of digital design and production tools.

5.1 3D2REAL | Cairo Shell

5.1.1 Exhibition Stand 3D2REAL

The project 3D2REAL was initially conceived in 2009 as an exhibition stand for the design store MAGAZIN at the Blickfang Design Fair (for Furniture, Fashion and Jewellery) in Stuttgart. The stand consisted of a wall system, serving as a filter between the design objects on display and the visitors. Planar elements, arranged in a honeycomb-like grid layout, were oriented at specific angles to guide the observers' view to the objects behind the wall while shielding other areas from sight. This was achieved by aligning the elements toward five focal points, with each design object receiving its own focal point (ref. Figure 6).
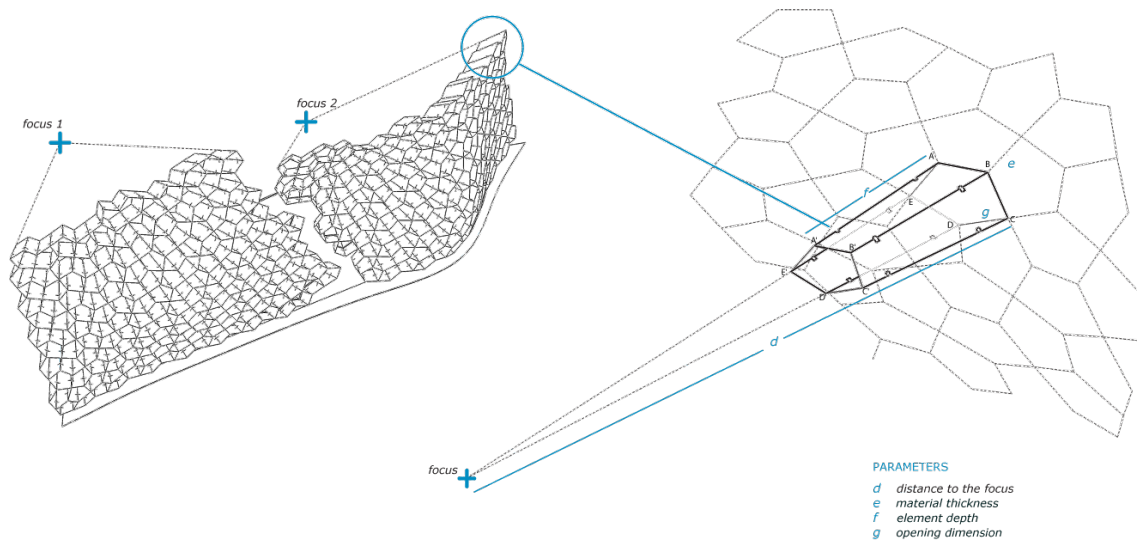
PARAMETERS
d distance to the focus
e material thickness
f element depth
g opening dimension

Figure 6: 3D2REAL Concept to generate specific visual relationships

Behind the wall, the opposite effect was achieved - the view to the outside from the focal points was completely unobstructed, allowing a panoramic perspective as the honeycomb elements were aligned perpendicular to the observer's eye (ref. Figure 7)
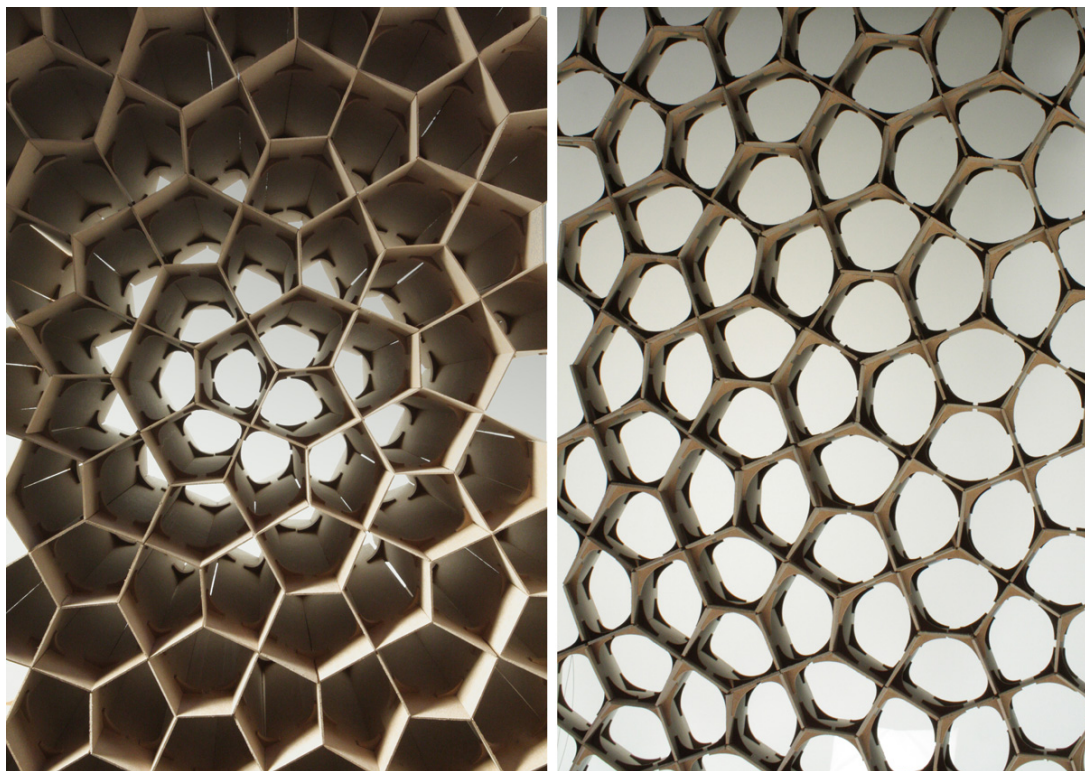

Figure 7: 3D2REAL Wall Section – View from outside in and from the inside out

The pattern chosen for the grid layout was a Cairo tiling. While aesthetically pleasing, this pattern is highly kinematic when pin-jointed (see also Section 5.1.2). To stabilize the system for the wall, a joint connection detail that prevented rotation was therefore chosen (ref. Figure 8).
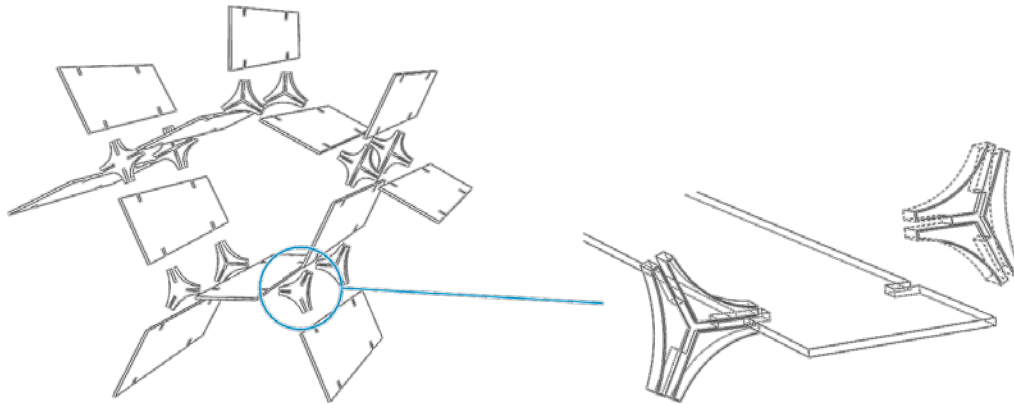
Figure 8: 3D2REAL Construction detail for the planar elements

Due to the nature of the structure and the overall free-form geometry, the system consisted of highly variable part geometries. In fact, of the over 2000 planar elements, no two were identical. As such, the structure had to be designed using a parametric approach und produced with digital CNC fabrication technology, cutting each individual element geometry from 3 mm MDF sheets.

The result, shown in Figure 9, was a complex structure with high architectural appeal, achieving the visual effect of focusing the visitor's attention to the exhibition objects as intended.
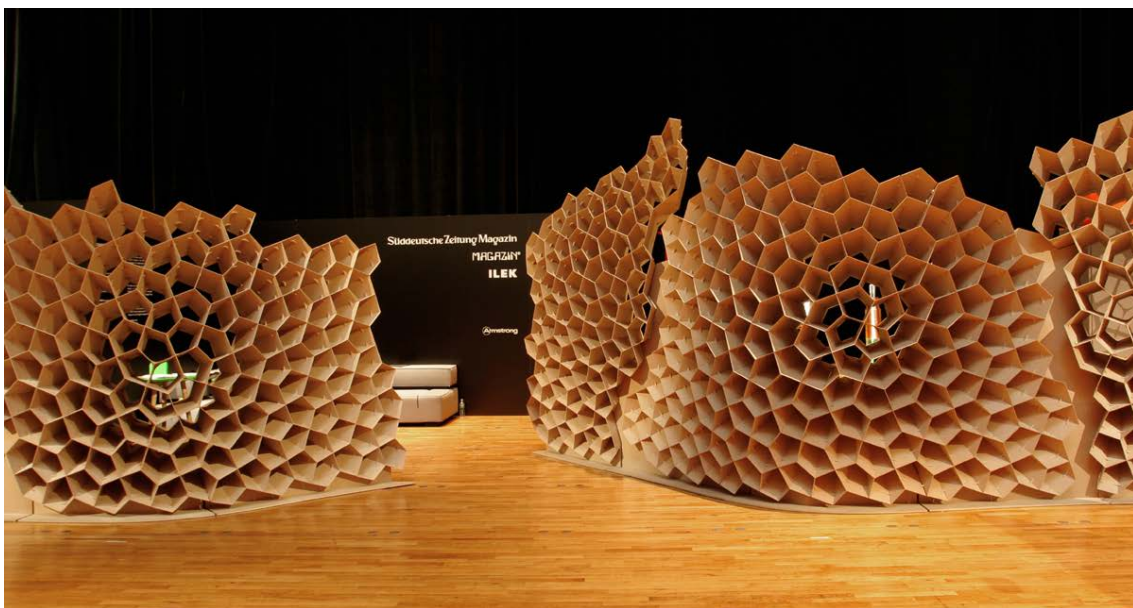
Figure 9: Exhibition stand 3D2REAL

5.1.2 Cairo Shell

To further investigate the potential of Cairo tiling as a structural system, a proposal to build a dome type shell structure was investigated (ref. Figure 10). The Cairo tiling pattern consists of pentagons, and it is kinematic when pin-jointed. Using the tools described in Section 2.1 studies were performed to determine the actual degree of kinematic indeterminacy as a function of element numbers (ref. Figure 11). The repetition factor in this study indicates the number of repetitions of the basic element arrangement applied in each dimension (repetition factor 1 shows the basic arrangement). The total number of grid elements is therefore proportional to the square
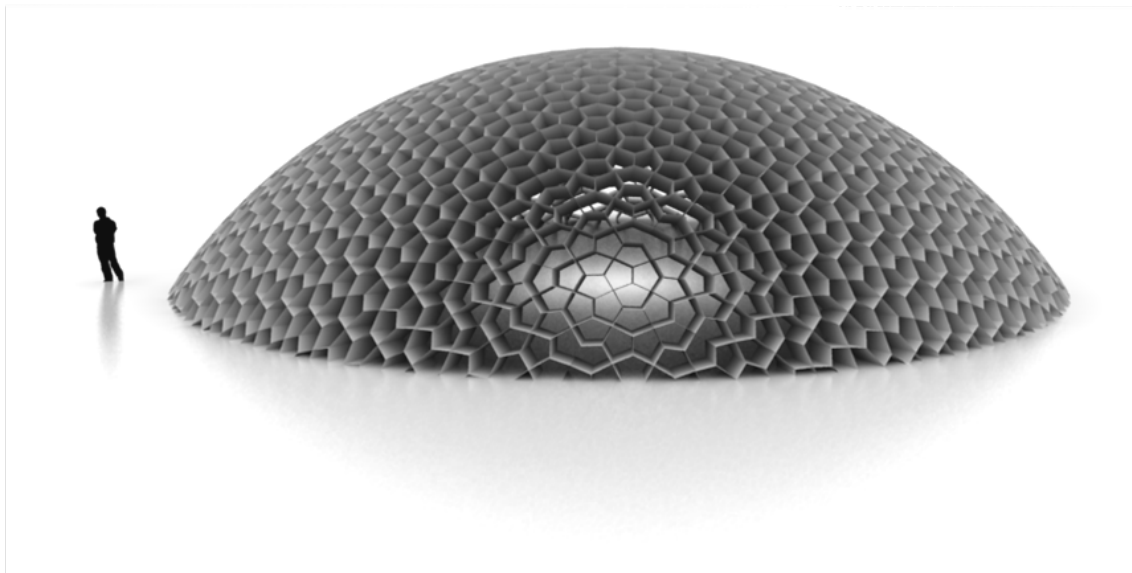
Figure 10: Cairo Shell Proposal

of the repetition factor. It is reasonable to expect that the degree of kinematic determinacy (calculated as the difference between rank and number of rows of the equilibrium matrix) is also proportional to the square of the repetition factor. The results in Table 1 confirm this expectation.
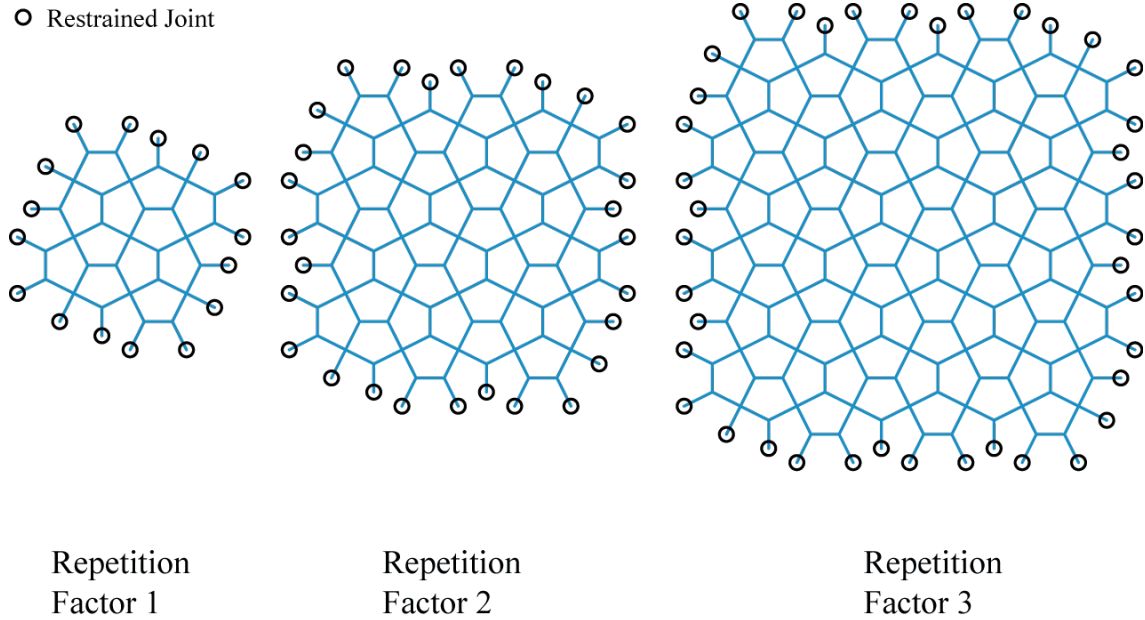
○ Restrained Joint

| Repetition Factor 1 | Repetition Factor 2 | Repetition Factor 3 |

Figure 11: Static determinacy properties of pin-jointed Cairo Tiling in 2D (ref. Table 1)
(Note: Repetition Factor 4 not shown)

Table 1: Static Determinacy Properties of Cairo Tiling

Equilibrium Matrix	Repetition Factor (ref. Figure 11)			
	1	2	3	4
Rows	58	162	314	514
Columns	56	148	280	452
Rank	54	146	278	450
Kinematic Indeterminacy	4	16	36	64

In the generation of the 3D structure for the Cairo Shell, the grid pattern is extruded to a focal point. Because of this extrusion, the static and kinematic properties in the plane of the pattern are exactly the same for the 3D system as for the 2D system. The analogy is shown in Figure 12 where it is evident that the kinematic deformation behavior of the rectangle in 2D corresponds to that of the 3D system.
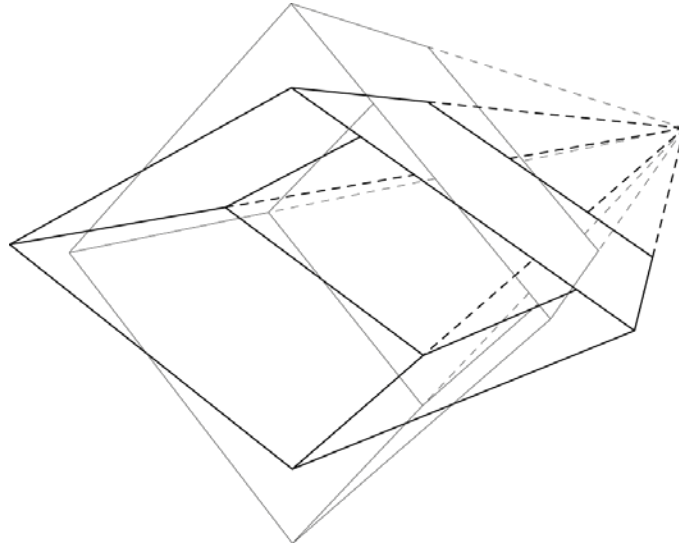
Figure 12: Kinematic behavior of point extruded pattern

Using a dxf-file to digitally transfer the geometry, additional investigations were performed using a plate element FEA model (in the software package SAP2000®, ref. Figure 13) to assess the behavior of the system with two different stabilization methods (ref. Figure 14):

a. Moment connections only, resisting relative rotation between the plate elements

b. With additional cross bracing elements in the open cells. The cross bracing in this analysis represented sections of 0,2 mm x 200 mm ETFE film (tension only) to simulate a continuous film.

The investigated shell shown in Figure 13 was a truncated sphere (26.5 m sphere diameter) with a base diameter of 22.0 m, and a height of 5.8 m. The honeycomb elements with a depth of 0.8 m consisted of 3 mm MDF plates. The initial investigation was performed with self-weight loading (0.07 kN/m²).

To compare the two systems, the deflection and the forces acting on a trapezoidal honeycomb element at the shell vertex were investigated. As shown in Figure 15 these elements were subject to an axial force *P* (indicating membrane action) as well as a moment *M* (indicating bending action).
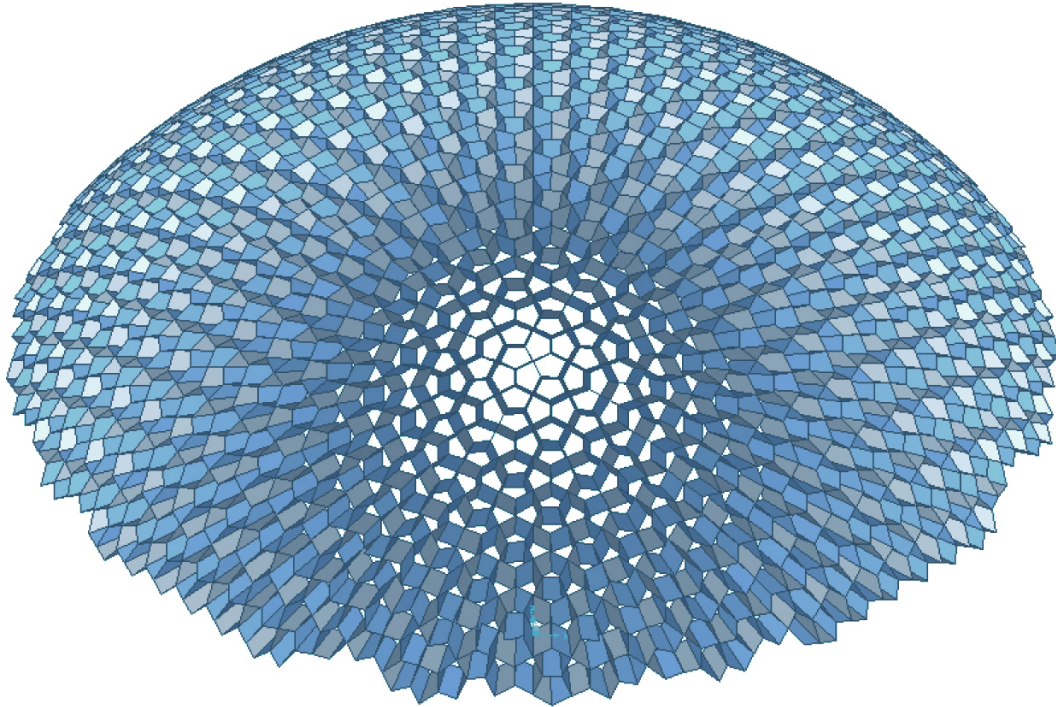
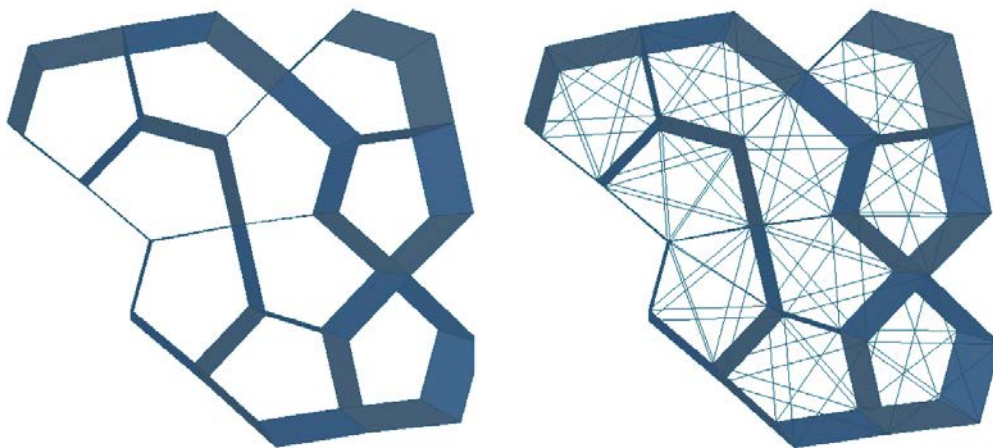*Figure 13: Structural Model of Cairo Shell*

Figure 14: Modeled structural systems: with moment connections, with cross bracing
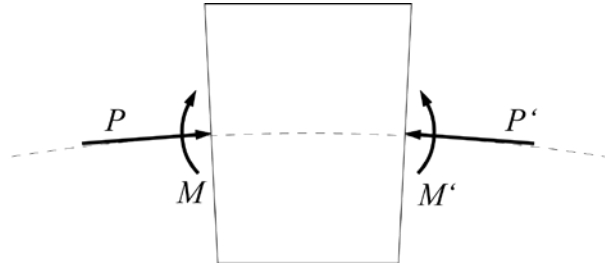
Figure 15: Forces acting on honeycomb element of Cairo Shell

As expected, the results of the preliminary analysis (ref. Table 2) show an increase in overall stiffness with the bracing. In addition, there is an increase in membrane action (indicated by the force *P*) and a decrease in bending action (indicated by the moment *M*).

Table 2: Results of preliminary investigation of Cairo Shell structural system

System	Deflection at crown (mm)	Force *P* at crown (kN)	Moment *M* at crown (kN·m)
Moment connections only	3,5	-155	74
Additional bracing	2,7	-218	48,2

A novel and very elegant method of attaching an ETFE film as a bracing system onto the grid structure would be by the application of a vacuum to draw a layer of film both to the inside and the outside surface of the shell. The pressure difference would generate a clamping force between the film and the main grid structure, resulting in friction that in turn would be relied upon to transfer the stabilizing forces from the grid system to the continuous film.

Vacuumatic structures have been realized successfully in the past (ref e.g. [8]), and initial experiments on 2D grids showed great potential. However, applying this method to a doubly curved 3D structure generates additional complexities. The inside and outside surfaces of a shell, while being subject to the identical vacuum pressures, do not have the same area. Thus, a net down force is generated when the vacuum is applied. This is seen in Figure 16 for a small group of elements, where the sum of the normal forces on the outer shell surface is greater than those on the inner surface.
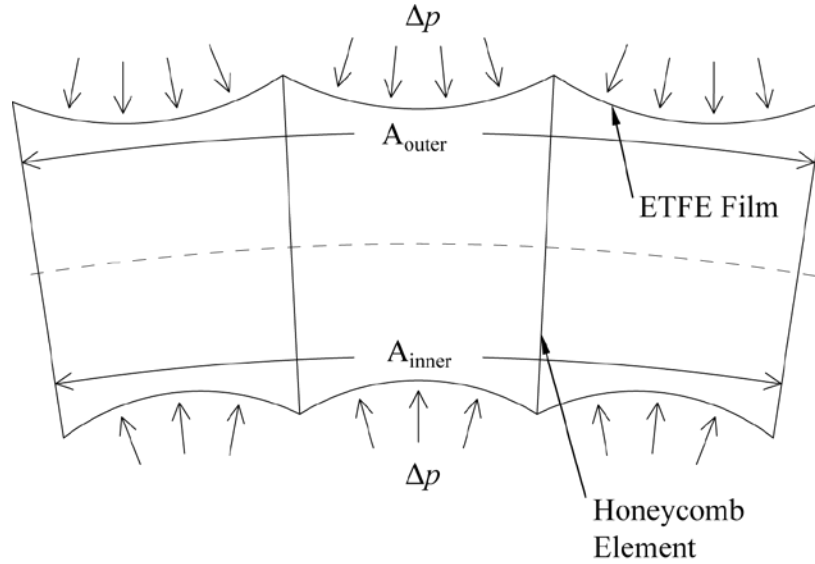
Figure 16: Pressure differential acting on curved structure

Because the applied forces are proportional to the surface area, the overall net force (acting normal to the shell surface) can be calculated as

$$P_{Tot} = \Delta p \cdot (A_{outer} - A_{inner}) \qquad (4)$$

This force is distributed uniformly over the shell surface.

Analysis has shown that for the shell structure proposed above, a frictional force (between the ETFE film and the main grid structure) in the range of P_{Stab} = 250 N at each joint is required to stabilize the structure when subject to self-weight. Using simple experiments the coefficient of friction between MDF and ETFE was determined to be μ = 0.25. Based on the tributary area for each node ($A_{Trib} \sim 0.25$ m²) and

$$P_{Stab} = \Delta p \cdot A_{trib} \cdot \mu = 250N \qquad (5)$$

a pressure difference of Δp = 0.04 bar can be calculated as the sufficient vacuum to achieve the required frictional force.

Using this pressure difference in Equation (4) with the dimensions of the proposed shell results in a net force P_{Tot} = 272 kN. This force is distributed evenly across the surface, resulting in a net pressure of 0.55 kN/m² acting normal to the shell surface. This load is much greater than the investigated load of selfweight (0,07 kN/m). With the additional net pressure, an even greater stabilizing force P_{Stab} would be required, necessitating an even greater pressure Δp, and thus establishing a cycle of ever increasing stability force

requirement and pressure. This puts into question the benefit of the vacuumization method for stabilization purposes in the first place, at least for this particular system.

The following questions regarding vacuum stabilization warrant further investigation:

- Under which dimensional conditions (such as curvature and element depth) is vacuum stabilization a viable option?

- What is the difference in the stabilization effect in the simple model with nodal bracing (ref. right hand side in Figure 14) compared to the stabilization effect truly achieved by application of the film (acting mostly along the edges of the grid elements)?

- What is the effect of the "clamping force" generated by the vacuumized film on the elements with respect to plate buckling?

Additional research is currently underway to address some of these questions and confirm the above findings, including non-linear FEA models of the combined system (grid structure and ETFE film) as well as investigations into other methods to stabilize the structure.

5.2 rn601

Following the success of the 3D2REAL exhibition stand, a new installation was realized for the 2010 Blickfang fair. Serving as a unique framework for the design and furniture objects selected by MAGAZIN, a foam-like structure of varying density was conceived. This structure took the previous application of complex grid patterns from a quasi-planar system to a true 3D spatial structure. The individual cells, initially based on a Weaire-Phelan-Structure were subsequently deformed through computational algorithms to specifically draw attention to the displayed objects (ref. Figure 17).
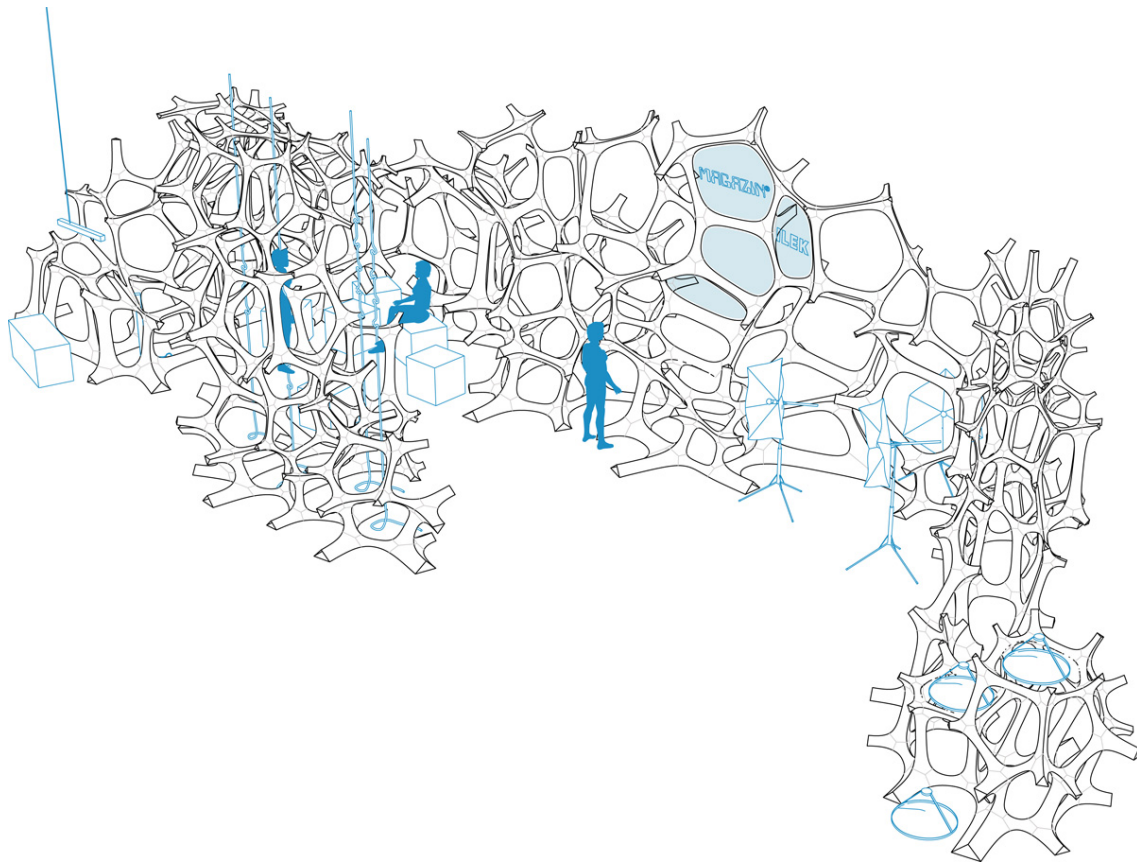
Figure 17: Overall view of rn601 structure

The parametric deformation approach allowed the topology of the original regular structure to be retained (such as four members connecting at each joint) and took into account material properties (for example minimum bending radii) and production constraints (for example maximum element size) during the design process. One important restriction was that the structure needed to consist of developable 2D elements in order to allow for an economical milling fabrication process. The initially planar elements were then assembled in a 3D puzzle-like fashion (ref. also [9]) to form a spatial system. To facilitate an efficient joining technique an elaborate interlocking pattern was introduced at the element edges immediately prior to the production phase (ref. Figure 18 and Figure 19). The final structure consisted of over 5000 individual pieces, all of which were unique in their geometry and produced from 2mm PVC foam board panels using CNC milling technology. Despite being subject to rigorous algorithms and conditions, the result is a seemingly freeform spatial structure of great architectural impact (ref. Figure 20)
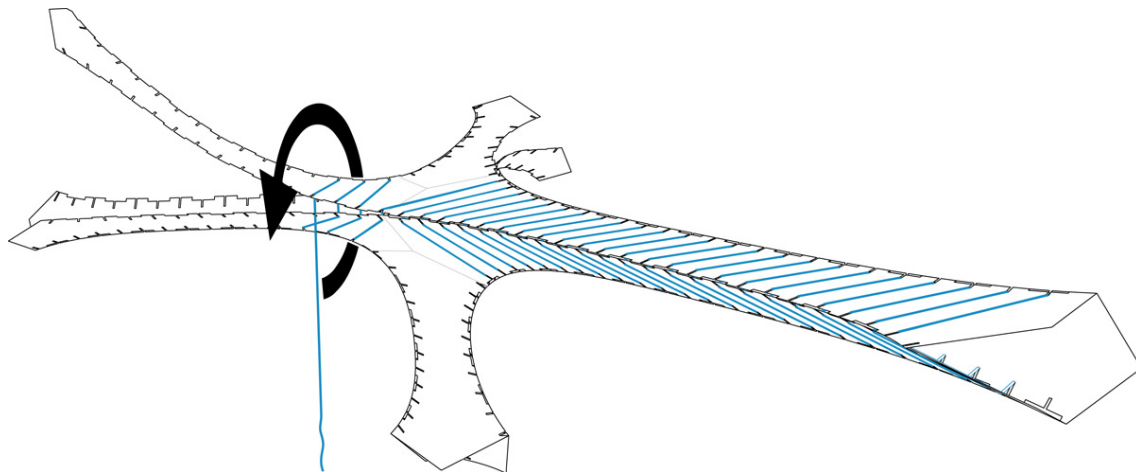
Figure 18: Node detail for rn601

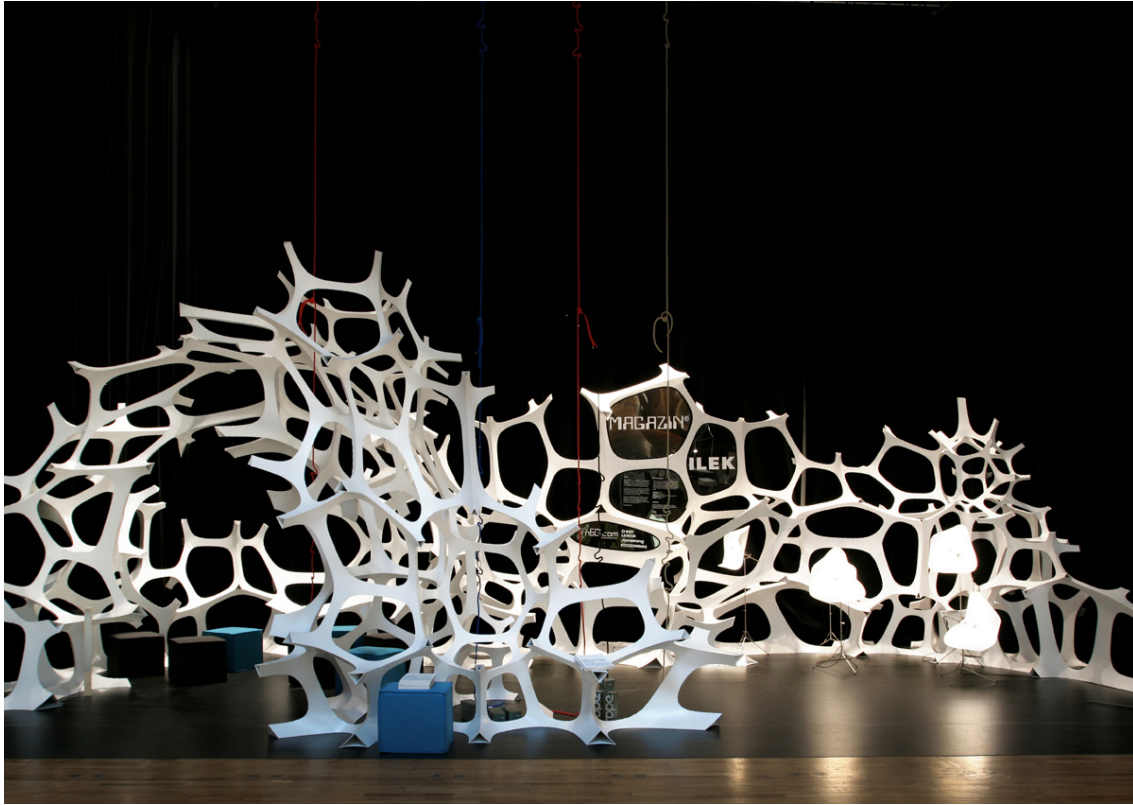*Figure 19: Assembly detail for rn601*

Figure 20: Completed rn601 structure

The final structure was extremely light-weight (< 150 kg). Structural analysis was performed to determine the material stresses of the system when subject to self-weight. The geometry was exported from Rhinoceros® in IGES-format (which can handle curved NURBS surfaces) and imported into the FEA software package Ansys® for analysis (ref. Figure 21). As expected, the results showed that for the given load case of self-weight only, the material stresses were very small (< 200 kPa peak von Mises stresses).

Transferring geometry using a compatible file format as described in the two case studies above is only the first step towards an automated integrative process. In a next step, software tools can be written to automatically generate input files for analysis software, execute the solution, and read and process the results. As such, the structural behavior, similar to results from other analyses (for example sunlight simulation, ref. Section 3) becomes part of a comprehensively informed model.
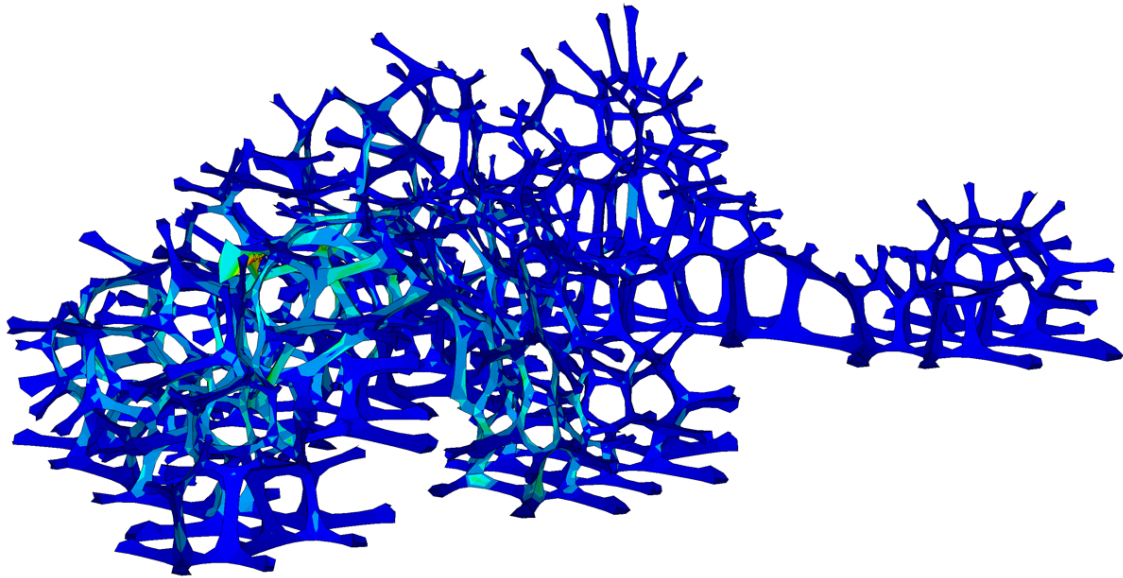
Figure 21: Structural analysis of rn601 structure

6 Conclusions and Outlook

The concepts outlined above and the related case studies show the complexities but also the great potential of complex grid systems. Computational design and production tools were implemented to generate structures that appear to be freeform, yet adhere to strict rules imposed during the design process. In addition to automating the consideration of parameters such as material properties and fabrication constraints using a parametric design approach, custom tools were developed to perform investigations into the structural and architectural performance of these complex systems.

The concept of vacuumization to stabilize a structure, although initially thought to be a very novel and efficient method, presented a number of complexitites upon closer investigation. Further research as outlined in Section 5.1.2 is necessary to draw comprehensive conclusions about this method.

The continued integration of the tools at the disposal of architects and engineers will allow the realization of highly differentiated and complex structures(grid systemsand otherwise). To facilitate the investigation and optimization of various performance criteria, additional tools will be created to interface design and analysis software with the goal of acquiring comprehensive information-based models. Most software packages already permit such an approach by providing programming interfaces. It is the authors' belief that the potential of parametric design and digital fabrication will be further enhanced by the automated integration of these tools.

7 Acknowledgements

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