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Validating Thrust Network Analysis using 3D-printed, structural models

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Abstract

Scale models can be used to understand the equilibrium of masonry systems – and structural compression forms in general– as they stand not because of allowable stresses, but because of their geometry. This paper demonstrates the use of 3D-printed, structural models, and emphasizes their relevance, not only to assess the stability of complex masonry vaults, but also to push the limits of new compression-only structures. Thrust Network Analysis (TNA) is an innovative approach for exploring three-dimensional funicular networks [1]. The TNA methodology was originally developed for stability analysis of historic vaulted structures in unreinforced masonry, extending graphic statics to fully 3-D problems. This framework is even more powerful as flexible approach for finding structural compression forms. Through the control of reciprocal force diagrams, which relate form and forces, new unexpected forms for compression-only shells become possible.

This paper will show and discuss some surprising compression forms obtained with TNA to demonstrate how the approach gives the designer the power to start exploiting structurally indeterminacy of three-dimensional funicular systems. Thanks to recent advances in TNA [2], which integrate the interactive structural form finding process with geometric and fabrication constraints into a smooth digital chain, discrete 3D-printed scale models can easily be produced. These unglued “masonry” scale models serve as very convincing first validations of the capabilities of this novel approach.

Keywords: Structural scale models, form finding, rapid prototyping, masonry vaults, funicular vs. freeform design

1 Introduction

Gothic cathedrals were built well before the introduction of structural theory [3]. The old master builders counted on experience and exceptional intuition –but also a lot of trial and error– to accomplish their stunning vaulted spaces in unreinforced masonry. They were furthermore also helped by a specific property of masonry structures. Unlike modern structures, not stress, but stability is of concern for this type of structures [4,5]. Because masonry has no (or very little) capacity to resist the induced tension due to bending, it has to be shaped such that it acts in compression only. If an unreinforced masonry structure does not have a good structural form, it will collapse. “Masonry does not lie”, and the stability of masonry structures is thus mainly a question of geometry, and not of material failure. In general, the stresses are low in structures which follow the flow of compressive forces, i.e. *funicular* structures, particularly when they are as bulky as most historic structures in masonry.

That the behaviour of masonry structures is only a matter of geometry, and not of stresses, makes their behaviour independent of scale. This powerful notion made it possible for the master builders to push the limits of imagination over centuries of evolution of form. They could use geometric rules (Fig. 1), which allowed them to copy the geometry of successful precedents and to scale them up; and scale models, which allowed them to check the stability of vaulted creations and to carefully balance them where necessary by adding blocks on the extrados.

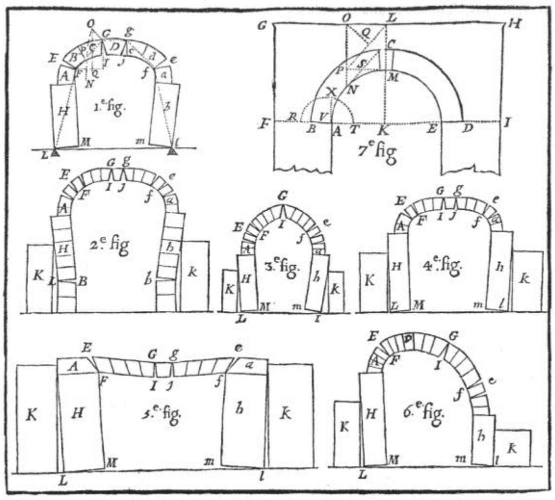

Fig. 1: Geometrical rules for stable arch-on-buttresses structures, and drawings of displacement studies with plaster models [6].

The stability of masonry structures is best assessed with limit analysis [3,7]. Simply put, an unreinforced masonry structure will be stable if within its section a compression-only system of forces can be found in equilibrium with the applied loads [8,9]. The applied loads are for masonry structures their dominant self weight. In order to be able to apply an equilibrium analysis (i.e. limit analysis) for the safety assessment of masonry structures, Heyman introduced three key assumptions [10]: masonry has no tensile capacity, sliding does not occur at the interfaces of the separate *voussoirs* (blocks), and masonry is considered rigid. These will be discussed further in the paper.

2 Learning from the old master builders

The understanding of the equilibrium of complex vaulted structures in masonry has been lost over the centuries. One is dazzled when realizing that the stunning fan vaults of the chapel of King’s College at the University of Cambridge, England have a structural stone shell with an average thickness–over-span ratio smaller than that of an egg shell. This becomes even more impressive when understanding that this structure is composed of thin stone slats held in place only by compression (and friction).

Thrust Network Analysis (TNA), a fully three-dimensional equilibrium approach, allows now to explain and visualize their stability by finding compression-only networks of forces, staying within the geometry of the structures [9]. Originally developed for the analysis of masonry vaults, this approach becomes more powerful as design tool. TNA allows the designer to carefully control three-dimensional funicular, i.e. compression-only, shapes. This flexible control results in a new formal language for stone structures.

The goal of this paper is to demonstrate the power of TNA for the design of novel shapes in unreinforced stone, and thus shells in general, which attempts to blur the boundaries between funicular and freeform design. A first, but convincing validation for this approach is given by discrete, unglued structural models with surprising shapes.

3 From abstract to model

An important challenge in realizing the structural models presented in this paper, is the transition from the abstract and discrete representation used for form finding, to the physical parts of the model. An example of a similar challenge is the translation of the hanging string model for the crypt of the Colonia Güell Church into an actual stone structure. It is Antoni Gaudí who was able to see form through these strings.

Using TNA, a thrust network is generated, comparable to the strings in Gaudí’s hanging model. In the form-finding process, several levels of control are provided (Fig. 3): form diagram, force diagram and loading (= self-weight) density. The form diagram defines the general outline of the structure, and force line topology, and the force diagram represents the equilibrium of the inner forces in the form diagram, or the horizontal force components of the resulting thrust network. Both diagrams enable the direct control by the user on form and the force distribution of the thrust network. The nodal self weight applied for the form finding process is controlled by the user as well.

By linking a continuous geometric representation, a NURBS surface, to the thrust network, tessellation and block generation independent from the given primal grid, is possible. Thanks to these recent developments in TNA [2], efficient design and production of the structural models presented in this paper became possible.

The structural behaviour of these unglued models is similar to the behaviour of unreinforced masonry vaults, because of the satisfactory fulfilment of Heyman’s three necessary requirements, i.e. the three assumptions introduced in Section 1:

- No tension: The compression-only geometry, generated by TNA, prevents tension forces to occurunder the (dominant) design loading.
- Rigid: The materials used for the 3D printed have very stiff material properties compared to the forces in the models. The funicular form, as a result of the formfinding using TNA, furthermore results in very low stresses, resulting in very small strains. The voussoirs can thus be considered as rigid.
- No sliding: Two conditions want to be satisfied to prevent this: the interfaces need to have “enough” friction (i.e. more than the Coulomb friction, or a friction angle of 0.6 or higher), and the stereotomy of the vault (i.e. how the vault volume is cut into

blocks) has to be such that sliding failure mechanism are prevented. The latter can be guaranteed by controlling the tessellation and individual block geometries. To provide sufficient friction between the model parts, the choice of rapid prototyping technique is crucial. Plaster-based 3D printers produce fairly rough pieces that have enough friction, which makes this technique perfect for the purpose of structural testing. A disadvantage of these printers is that geometrical tolerances seem not that reliable due to inaccuracies from e.g. shrinking. Further research is being done to control this process. Other rapid prototyping technologies are Fused Deposition Modelling (FDM) and Specific Laser Sintering (SLS), which are highly accurate in terms of geometry, but the surfaces of the thermoplastic material are very smooth and provide almost no friction. In order to prevent the interfaces to slide, small non-interlocking notches are provided at the interfaces of the voussoirs (Fig. 2). These were also helpful in the accurate registration of the pieces, controlling and simplifying assembly.

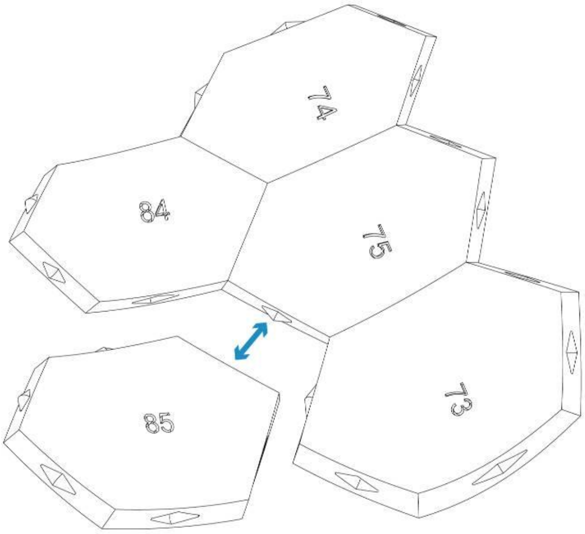

Fig. 2: Notches to avoid local sliding failure at the interfaces.

4 Designing a free-from masonry vault

In this section, the design process for a freeform, masonry-like vault model will be described. Using commercial NURBS modelling software, two input surfaces were built with one common edge, and two tangential edges. Using their local coordinate system, the form diagram, or *primal grid*, representing the choice of force lines in the vault and thus also the topology or planar projection of the thrust network was generated (Fig. 3a). From this, the reciprocal force diagram, or *dual grid*, is produced (Fig. 3b). The lengths of the branches in the dual grid represent the horizontal force components in the branches of the thrust network. Due to the structural indeterminacy of the primal grid, it is possible to deform the dual grid while maintaining certain rules [1]. An overlay of the information of primal and dual grid is the *force distribution*, represented by the pipe diagram in Figure 3c. The radii of the pipes are proportional to the axial forces in the branches. For a chosen self weight distribution, the *thrust network* is generated (Fig. 3d). The force distribution diagram (Fig. 3c) allows for a good interpretation of the resulting form of the thrust network, but also clarifies and visualizes that internal forces in the shell have to be redistributed in a certain way in order to achieve a specific three-dimensional effect. The thickest pipe segments on top of the diagram e.g. represent the forces in the most shallow edge arch (Fig. 6); the "loose ends" on the right side of the diagram represent the one continuously supported edge; or the backbone-like segments in the middle show the stepwise accumulation of forces in that region. This accumulation or channelling of forces results in the accretive kink in the thrust network towards the support.

Figure 6 shows some local changes of the tessellation in detail, in order to prevent sliding mechanisms, e.g. the geometry of the edge pieces (highlighted in blue) are constrained to but cut perpendicular to the edge arches.

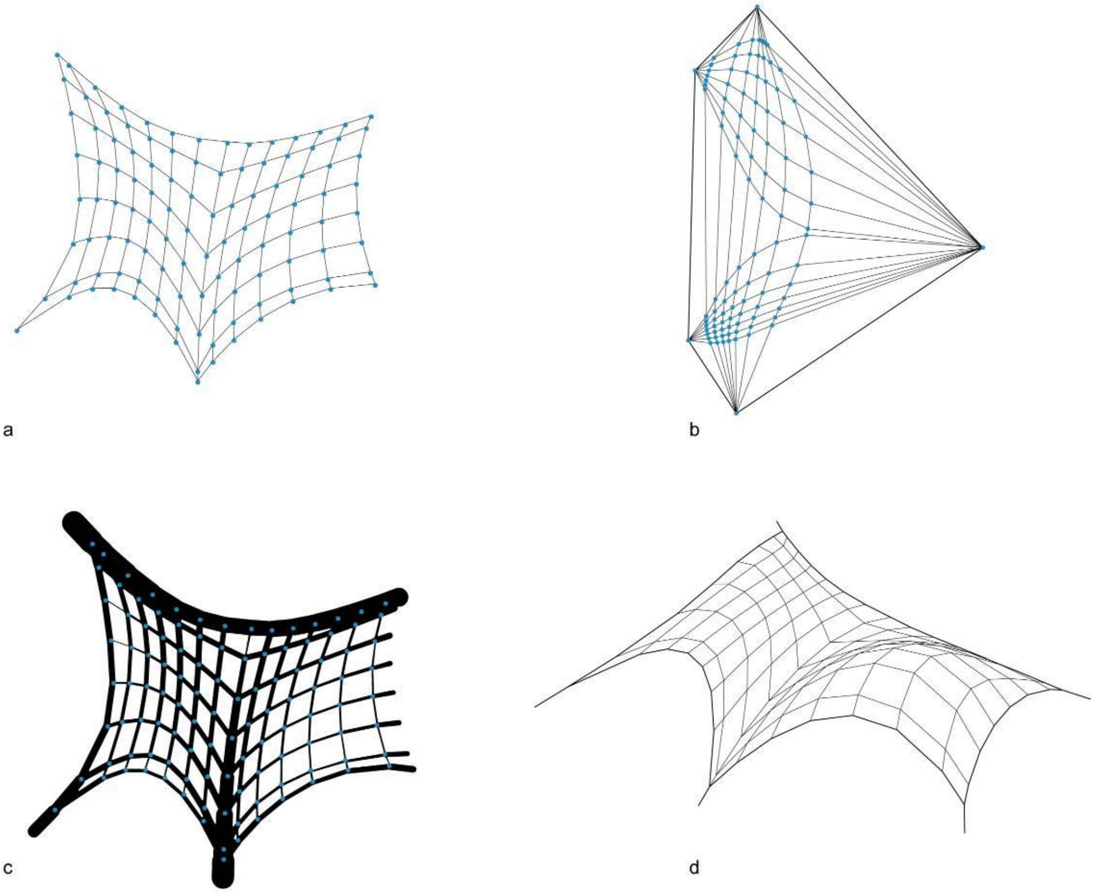

Fig. 3: The TNA form finding process of a "freeform" vault: a) the form diagram, generated from two "stitched" NURBS surfaces; b) its corresponding reciprocal force diagram; c) a pipe diagram, visualizing the force distribution; and d) the resulting compression-only thrust network in equilibrium with a given loading.

Fig. 4: Perspective view of the vault model.

Fig. 5: Perspective view of the vault model.

Fig. 6: Front view of the vault model.

Fig. 7: Top view of the vault model with marked edge tessellation pattern.

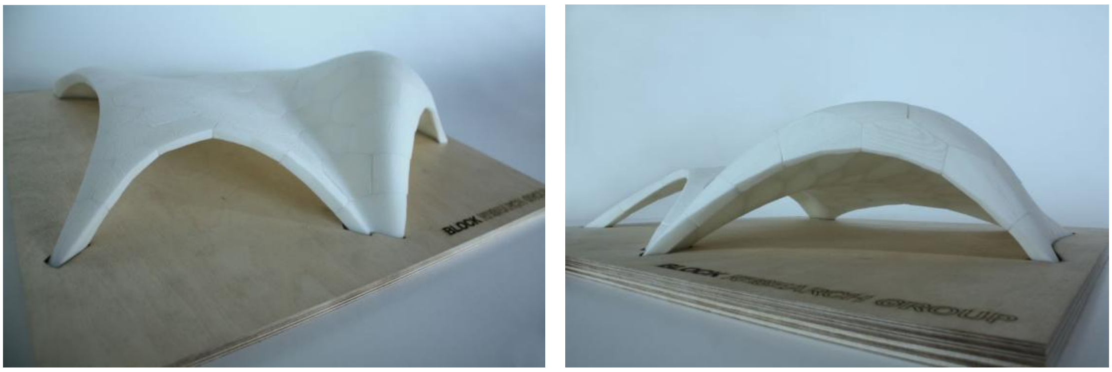

Fig. 8: Support details, showing the unglued pieces held together by compression.

5 Structural models as analysis tools

The current paper only showed the successful, first results of the capabilities of TNA by producing stable scale models of unreinforced masonry shells with novel shapes. This can be considered as a first validation of the approach.

More importantly, as the drawings of the tests with plaster models by Danyzy (Fig. 1, [6]) showed for two-dimensional structures, scale models can be used to investigate the stability and collapse mechanisms of discrete structures under e.g. support displacements or concentrated live loads (Fig. 9). This is a really hard, and not at all understood, problem for complex three-dimensional vaulted structures.

The use of scale models for sophisticated, three-dimensional structures, both historic and new, is now possible thanks to improved rapid prototyping technologies. The use of 3D printed scale models to further the understanding of collapse mechanism in 3D masonry has been first introduced by the first author, together with Prof. John Ochsendorf at MIT [13]. This approach has provided new insides into the stability of masonry domes [14,15]. Beyond the structural arguments for scale models, [2] gives additional reasons how scale models are invaluable design tools for new structures in masonry.

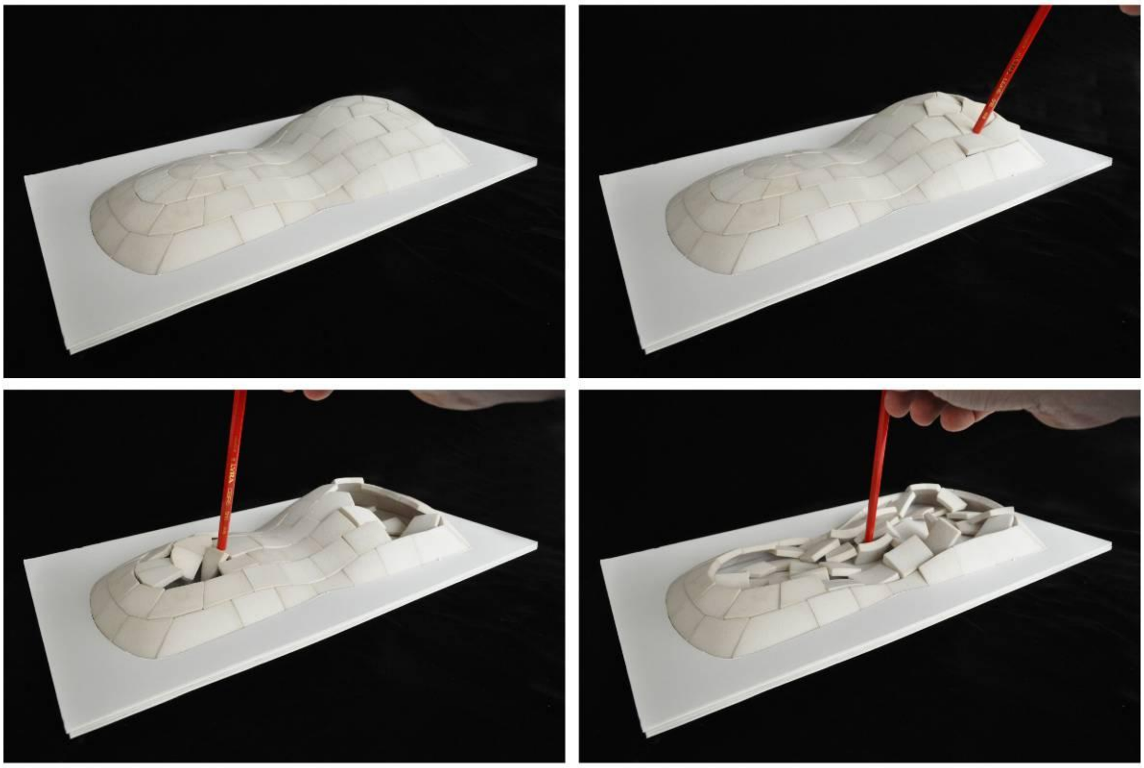

Fig. 9: Collapse sequence of a masonry vault model under point loads.

6 Conclusions

This paper showed new directions for masonry design possible thanks to new extensions to TNA. Rapid prototyping technology, combined with a customized digital workflow for the planning and production, has been successfully used to efficiently build structural masonry-like models. These models allow for a first validation and new insights in structural behaviour of such novel vault forms. Furthermore, due to the scalability of compression-only structures, they enable a reliable prediction of the behaviour of a real scale stone structure for corresponding load assumptions.

This paper demonstrated a prototypical case study for the design, production, and behaviour of 3d-printed models of masonry structures. Next research steps include the setup of a testing laboratory at the ETH Zurich, that will enable a systematic testing and analysis of masonry structures based on structural models.

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References

[1] Block P and Ochsendorf J. Thrust Network Analysis: A new methodology for three-dimensional equilibrium. *Journal of the International Association for Shell and Spatial Structures*, 2007; **48**: 167-173.

[2] Lachauer L, Rippmann M and Block P. Form Finding to Fabrication: A digital design process for masonry vaults. *Proceedings of the International Association for Shell and Spatial Structures (IASS) Symposium 2010,* Shanghai, China.

[3] Addis B. Building: 3,000 Years of Design, Engineering and Construction. London/NYC, Phaidon Press, 2007.

[4] Huerta S. Mechanics of masonry vaults: The equilibrium approach. In P. B. Lourenço and P. Roca (Eds.) *Proceedings of Historical Constructions* 2001. Guimaraes, 47-69.

[5] Huerta S. Arcos bovedas y cupulas. Geometria y equilibrio en el calculo tradicional de estructuras de fabrica. Madrid, Instituto Juan de Herrera, 2004.

[6] Heyman J. The Stone Skeleton: Structural engineering of masonry architecture. Cambridge, Cambridge University Press, 1995.

[7] Danyzy A. Méthode générale pour déterminer la résistance qu'il faut opposer à la poussée des voûtes. *Histoire de la Société des Sciences établie à Montpellier* 1732; **2**: 40-56.

[8] Block P, Ciblac T and Ochsendorf J. Real-time limit analysis of vaulted masonry buildings. *Computers and Structures* 2006; **84**(29-30): 1841-1852.

[9] Heyman J. The stone skeleton. *International Journal of Solids and Structures* 1966; **2**: 249-279.

[10] Block P. Thrust Network Analysis: Exploring Three-dimensional Equilibrium. PhD Dissertation. Massachusetts Institute of Technology. Cambridge, 2009.

[11] Chilton J. Finding Heinz Isler – The Engineer’s Contribution to Contemporary Architecture, Thomas Telford Ltd., 2000.

[12] Addis B. A history of using scale models to inform the design of structures. In S. Huerta (Ed.) *Essays in the history of the theory of structures. In honour of Jacques Heyman* 2005. Madrid, Instituto Juan de Herrera, 9-44.

[13] Block P. “Equilibrium y’all” Blog: http://equilibriumstone.wordpress.com/

[14] Zessin J, Lau W and Ochsendorf J. Equilibrium of cracked masonry domes, *ICE Engineering and Computational Mechanics* 2010.

[15] Quinonez A, Zessin J and Ochsendorf J. Small-Scale Models for the Analysis of Masonry Structures. *Proceedings of the 7th International Conference on Structural Analysis of Historical Constructions - SAHC 2010*. Shanghai, China.