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RESEARCH

MayaVault—a Mesh Modelling Environment for Discrete Funicular Structures

Vishu Bhooshan[1] · David Reeves[1] · Shajay Bhooshan[1,2] · Philippe Block[2]

Abstract
The didactic and fabrication related value of a geometric form-finding method such as Thrust Network Analysis(TNA) has already been established through RhinoVault. However, it also serves as the origin of our proposition that the objectives of the TNA method are better served within a Mesh Modelling Environment (MME). The main contributions of the paper are:

1. Incorporating geometric stiffness based form finding methods within an MME.
2. Articulating the nature and specifying from a designer's standpoint the requirements of an interactive exploratory MME that methods like TNA explicitly intend to support and
3. Exemplification of the downstream, usually fabrication related, benefits of combining TNA with an MME.

The paper also describes the current status of the authors' efforts in the development of a custom software add-in to the MME of Autodesk^ Maya to demonstrate the said benefits.

Keywords Computational form-finding · Force density method · Thrust network analysis · Architectural geometry · Mesh modelling environment · Mesh data-structures

✉ Vishu Bhooshan
vishu.bhooshan@zaha-hadid.com

David Reeves
david.reeves@zaha-hadid.com

Shajay Bhooshan
shajay.bhooshan@zaha-hadid.com

Philippe Block
block@arch.ethz.ch

1 Computation and Design Research Group (ZH CODE), Zaha Hadid Architects, London, UK

2 Block Research Group, Institute of Technology in Architecture, ETH, Zurich, Switzerland

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Introduction

The recent and rapid advances in the field of computational form-finding and gaining popularity of *architectural geometry* (Veltkamp 2010; Janssen 2011; Shepherd et al. 2011; Bhooshan 2017; Louth et al. 2017) have brought the principal stakeholders in the architectural design process—architects, engineers and fabricators, and their respective tool-chains much closer together. This is true both in the early design phase and across the design to physical production pipeline. The paper describes and prototypes a Mesh Modelling Environment (MME) for creating fabrication aware design geometry with an explicit aim of "incorporating essential aspects of function, fabrication and statics into the shape modelling process" (Jiang et al. 2015; Tamke 2015).

MMEs, as we define them, are digital environments that inherit certain specifications from tool-kits commonly found in the computer animation industry and enable the creation and manipulation of discrete representations or meshes. Such a tool-suite includes, but are not restricted to,

- A mesh data structure, usually a half-edge or winged-edge mesh;
- Conway operators (Hart 2006) such as chamfer of vertices, bevel of edges, extrusion of faces etc.;
- Storage structures for attributes on vertices, edges and faces of a mesh, along with algorithms to fair the data thus stored;
- A brush-based toolkit to interactively 'paint' attributes into the storage structures above;
- A dependency graph structure, along with persistent of data across the graph, to allow dataflow programming; and
- Constraint solver such as a projection guided solver (Bouaziz et al. 2012).

The use of discrete representations, which are an intrinsic part of an MME, though ubiquitous in computer graphics and animation industry, has hitherto been not as prevalent in architectural design. This is mostly due to the lack of appropriate creation and manipulation tool-sets in popular Computer-Aided-Design (CAD) applications used by architects (Pottmann et al. 2006). Aided by recent developments in the application of discrete differential geometry to architectural design problems, the paradigm of surface based architectural design geometry predominantly favours a mesh based representation (Prévost et al. 2013; Panozzo et al. 2013; Schwartzburg and Pauly 2013).

Computational form-finding methods used in the design of funicular structures can be categorised in three main families (Veenendaal and Block 2012):

1. Stiffness matrix methods are dependent on the choice of material to be used in the physical manifestation, and usually employ finite element methods (FEM) and optimisation techniques. For design applications of these methods we refer the reader to Søndergaard and Dombernowsky (2012); Nahmad Vazquez et al. (2014); Markopoulou et al. (2016); Bhooshan et al. (2017); Louth et al. (2017).

2. Geometric stiffness methods are better suited for early design exploration, when material is generally not known and the designer is only able to provide simple input parameters such as boundary conditions, preset design loads etc. (Rippmann 2016). Popular methods in this family include the force density method (FDM) and thrust network analysis (TNA). For design applications of FDM (see Happold and Liddell (1975); Linkwitz (2014)) and TNA (see Rippmann et al. (2016)).
3. Dynamic equilibrium methods, like geometric stiffness methods, are suited for early design explorations. Dynamic relaxation (DR) and particle-spring systems (PS) are popular examples of the dynamic equilibrium family. For design applications of DR (see Harris and Roynon 2008; Adriaenssens et al. 2014a) and PS (see Bhooshan and El Sayed 2011; Bhooshan et al. 2014).

It can be noted that an MME along with an appropriate application program interface (API) fulfils the 'recipe' for form-finding algorithms as described by Veenendaal and Block (2014), especially the geometric stiffness and dynamic equilibrium methods.

Contributions and Organisation

This paper describes the design-exploration and downstream geometry processing benefits of incorporating geometric stiffness methods— particularly TNA—within an MME such as Autodesk® Maya. The main contributions of the paper are

1. specifying the software requirements to support interactive exploration of structurally sound shape and
2. the exemplification of the downstream benefits of combining TNA with an MME.

"Relevant Prior Work" summarises previous work as related to embedding form-finding methods in partial MME, and the extension of an MME to incorporate fabrication related constraints. "Shape Design and Network Analysis" describe the software specification for interactive form-finding and exemplifies the *designerly* (Cross 2001) benefits of the MME. In "Discussion" we discuss the downstream benefits of integrating TNA in an MME and describe the possibilities of establishing a feedback loop between fabrication constraints discovered downstream back into upstream procedural topology creation and form-finding process. In addition we describe the current implementation of MayaVault and conclude in "Conclusion".

Relevant Prior Work

RhinoVault (Rippmann et al. 2012b), as a prototype implementation of TNA in the non-MME environment of McNeel Rhinoceros®, is currently the only publicly available implementation. The algorithmic benefits of using a mesh data-structure for TNA, including the extensions to subdivision schemes and interactivity potentials are already noted by several researchers (Vouga et al. 2012; Tang et al. 2014). The design-exploration and shape design benefits of embedding dynamic equilibrium methods within an MME has also been noted previously (Bhooshan et al. 2014). The benefits of MME, including creation and manipulation toolsets, subdivision schemes etc. in architectural shape creation and form-finding have been previously established (Shepherd and Richens 2010; Bhooshan and El Sayed 2011). There have also been prior attempts to combine them with DR-based techniques to address fabrication related constraints of realising them with fabric (Bhooshan and El Sayed 2012), curved-crease folded metal (Bhooshan et al. 2015b; Louth et al. 2017) and 3D printing (Bhooshan et al. 2017).

Feedback between design exploration and rationalisation in the context of funicular structures is discussed in detail by Rippmann (2016). Here the effects of TNA in mitigating the need for post-rationalisation are emphasised, as the method constrains the design exploration to a space of structurally relevant shapes (i.e. funicular under self-weight). However, an intermediate step between design exploration and production is still maintained in the described workflows wherein the resulting funicular surface is segmented into a set of force-aligned elements which are verified for stability and ease of fabrication (Rippmann 2016). MMEs, as noted previously in section "Introduction", are increasingly relevant for later stage design rationalisation tasks. Meshes are quickly becoming the preferred geometric representation at both ends of the design pipeline,with an MME presenting a significant opportunity for more holistic modes of design exploration—those which negotiate a broader range of design criteria (structural, fabrication, assembly etc.) within a unified form-finding process.

In the case of funicular structures, Vouga et al. (2012) and Tang et al. (2014) have made significant contributions in representing fabrication constraints within an interactive funicular form-finding workflow. Here, a projection-based scheme is used to satisfy a variety of production-relevant geometric constraints—in particular, those related to construction from flat sheet material—while simultaneously solving for static equilibrium. By allowing such production constraints to participate within the form-finding process, post-rationalisation can be circumvented as design exploration is limited to the space of self-supporting forms that are inherently feasible from a production standpoint.

While the importance of real-time user interaction is acknowledged by Vouga et al. (2012), it is limited, in this case, to geometric manipulation. This considerably narrows the scope of shape exploration as topology is predetermined and remains constant throughout the design process. The significance of topological modification during shape exploration in the context of MME is discussed by Bhooshan and El Sayed (2011). Furthermore, the relevance of coupling topologically dynamic data structures with digital form-finding methods in the context of early stage design exploration has been detailed and demonstrated by Deleuran et al. (2016) and Suzuki and Knippers (2018). We see this as an area where the MME integration of TNA could offer significant advantages as MMEs typically favour robust mesh representations that are well-suited to high-frequency topological modification and user interaction. Such data structures could potentially enable a far less deterministic design exploration of fabrication-aware funicular structures.

Shape Design and Network Analysis

Numerical form-finding techniques using *network analysis* such as FDM by Schek (1974) and TNA by Block and Ochsendorf (2007) have been well researched and documented. For comprehensive understanding of

- FDM, we refer the reader to Linkwitz (2014); Veenendaal and Block (2014), extensions of the method for an nonlinear approach Linkwitz and Veenendaal (2014), application to prestressed /tensegrity structures Zhang and Ohsaki (2006); Miki and Kawaguchi (2010).
- TNA, we refer the reader to Block (2009); Block et al. (2014); Veenendaal and Block (2014), interactive design applications using TNA Rippmann et al. (2012a, 2016); Rippmann (2016).
- the close algorithmic relationship between FDM and TNA, we refer the reader to Block (2009); Adriaenssens et al. (2014b).
- the distinct didactic and design-exploration benefits of TNA in relation to DR, we refer the reader to Rippmann (2016).

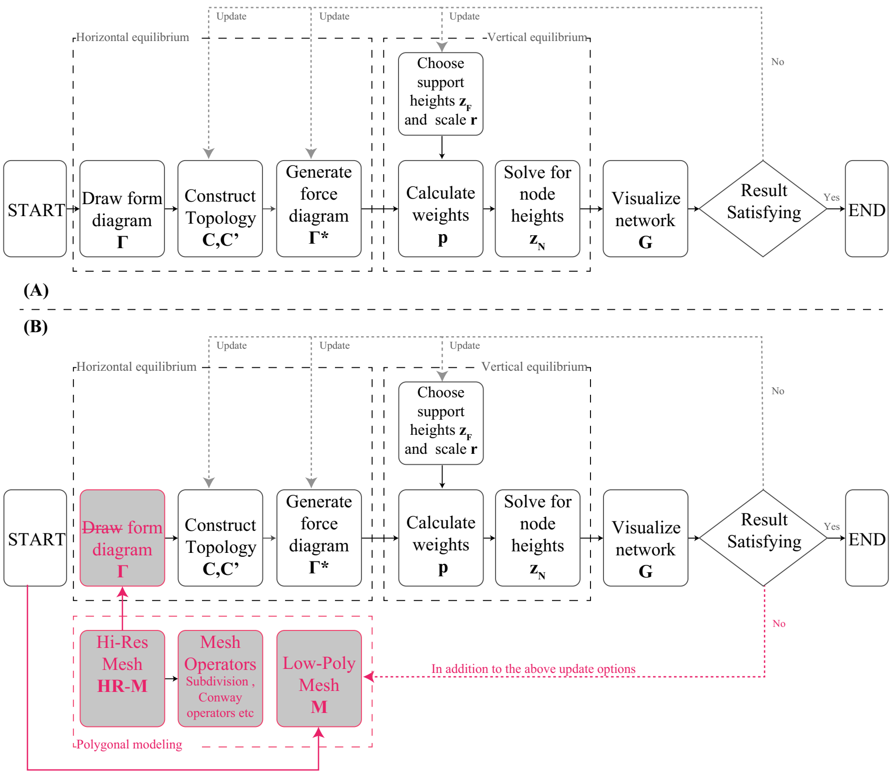

Fig. 1 **a** TNA workflow described by Block et al. (2014) and **b** modified TNA design workflow with mesh modelling techniques

The integration of network analysis in an MME stems from the authors pedagogic experience of implementing a modified TNA design workflow at a 10-day design to production workshop. It builds upon the TNA workflow described by Block et al. (2014) (Fig. 1a) by introducing a component of shape modelling in an MME to produce the *form diagram* (Oval et al. 2017, 2018) (Fig. 1b). The remainder of the design exploration workflow was carried out using RhinoVault. An overview of the results of such a workflow are presented in Fig. 2. Further, the first three authors of this paper have previously developed a test-bed software add-in to incorporate FDM within the MME of Autodesk® Maya. This sandbox implementation also incorporates the use of polyhedral stress potential fields as described by Vouga et al. (2012) (Fig. 3).

Network Analysis, as noted by Veenendaal and Block (2014), requires a *control mechanism* through an User Interface (UI) model to have an intuitive understanding of the parameters—boundary conditions, topology, loads—and

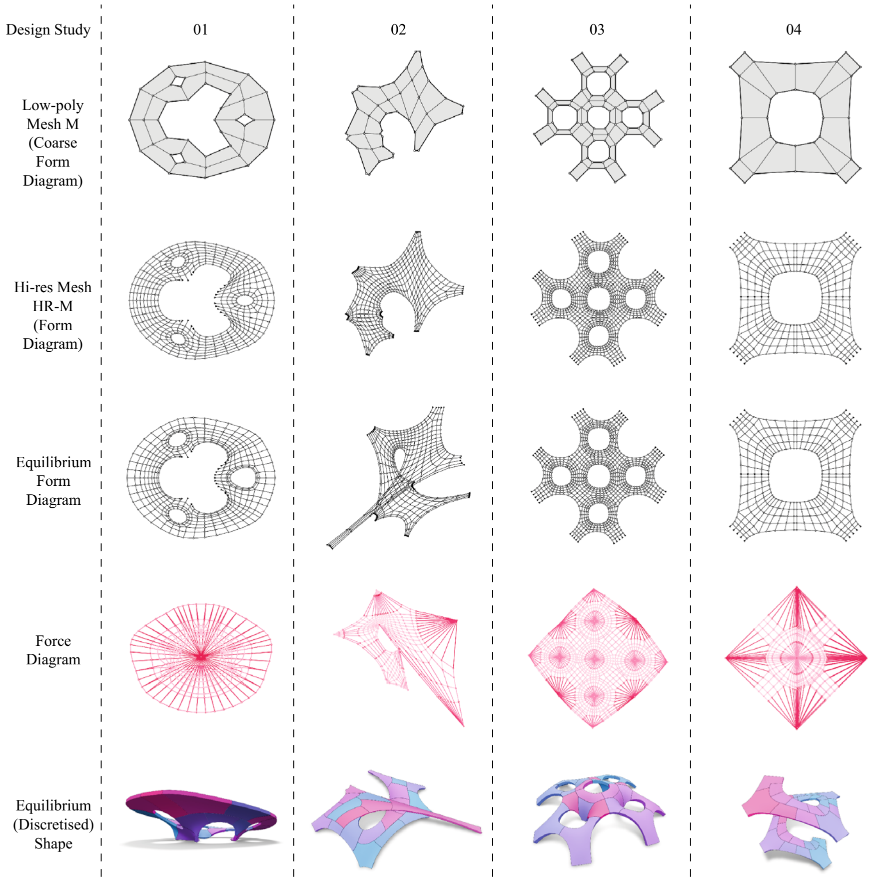

Fig. 2 Overview of results for mesh based TNA workflow

solver dependent parameters—*force densities* (Linkwitz 2014), *form* and *force diagram* (Block et al. 2014). An MME offers such a UI and incorporates the requirements for an interactive exploration of FDM and TNA. The advantages of such an environment with respect to early stage design exploration are:

1. Ease of creating a low-poly control polygon M. Such a manifold M, as noted by Bhooshan and El Sayed (2011), lends itself to easy manipulation of the input design geometry facilitating variation and quick iterations.
2. Ease of carrying out topological changes on M using Conway operators and building an understanding of their influence on the final result mesh (G-M).
3. Ease of updating/creating mesh vertex/edge/face attributes.

    (a) An MME brush toolkit allows for interactive assignment of element attributes via colour (vertex /edge colour attribute) (Fig. 3). Such a tool kit coupled with pairing of the edge colour attribute to that of the force density, provides for an interactive as well as a didactic value in understanding the influence of force densities on G-M (Fig. 4).

    (b) A mesh incorporates the advantages of the data structure noted by Veenendaal and Block (2014) allowing the designer to switch from FDM to TNA when direct control over both form diagram(Γ) and force diagram (Γ*) is required (Block et al. 2014). This gives the designer the ability to maintain certain design features in plan such as facade line by fixing the form diagram and manipulating the force diagram.

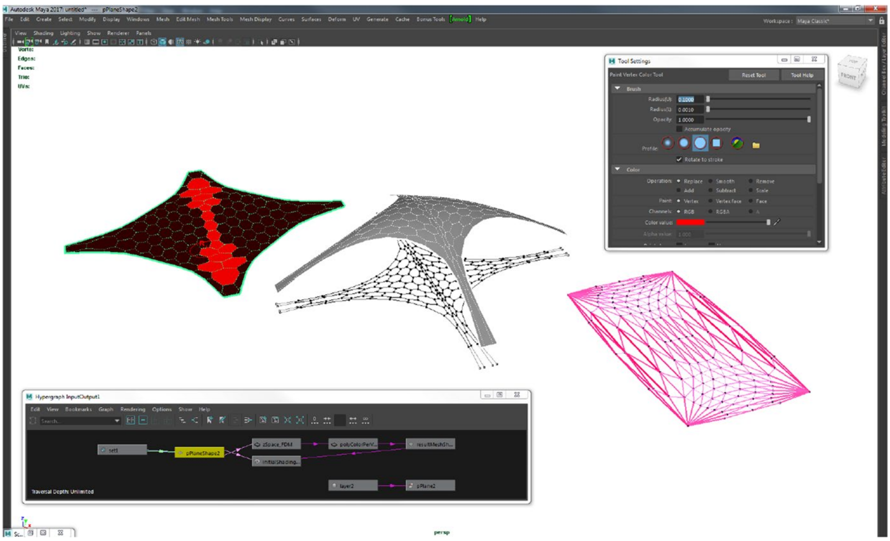

Fig. 3 Test bed software, incorporating FDM within the MME of Autodesk® Maya highlighting the brush toolkit to paint force densities, an *hypergraph* environment to maintain dependency graph structure in built in Maya. Additionally the software also produces the result mesh,form and force diagram (using an half-edge mesh data structure) for further inspection via TNA

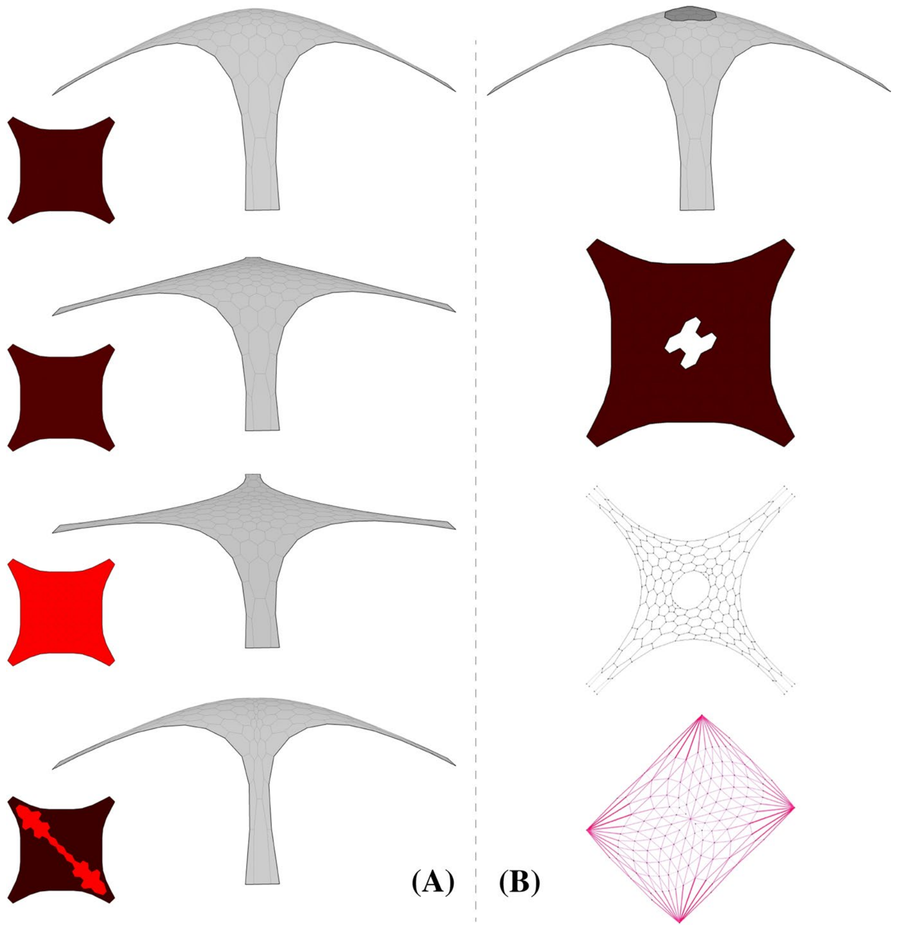

Fig. 4 **a** Intuitive understanding of effect of force densities on the final result via painting of vertex colour (implicitly edge colour) varying the red channel from 0 to 1. Higher arches are created for lower values of force densities i.e lower red channel value. The results are in agreement with those noted by Veenendaal and Block (2014). **b** Exploring the affects on final result, form diagram and force diagram by introducing holes in HR-M

4. An appropriate mesh data structure allows for the G-M, Γ and Γ* to be treated as different views of HR-M.

    (a) G-M, having the same topology as HR-M is stored as mesh vertex position attribute.
    (b) Γ, being the horizontal projection of GM, is also stored as additional mesh vertex position attribute.
    (c) Γ*, generated as per Maxwell's geometrical definition of reciprocal figures (Maxwell 1864; Block 2009), the edge lengths of the dual is stored as an mesh edge attribute of HR-M.

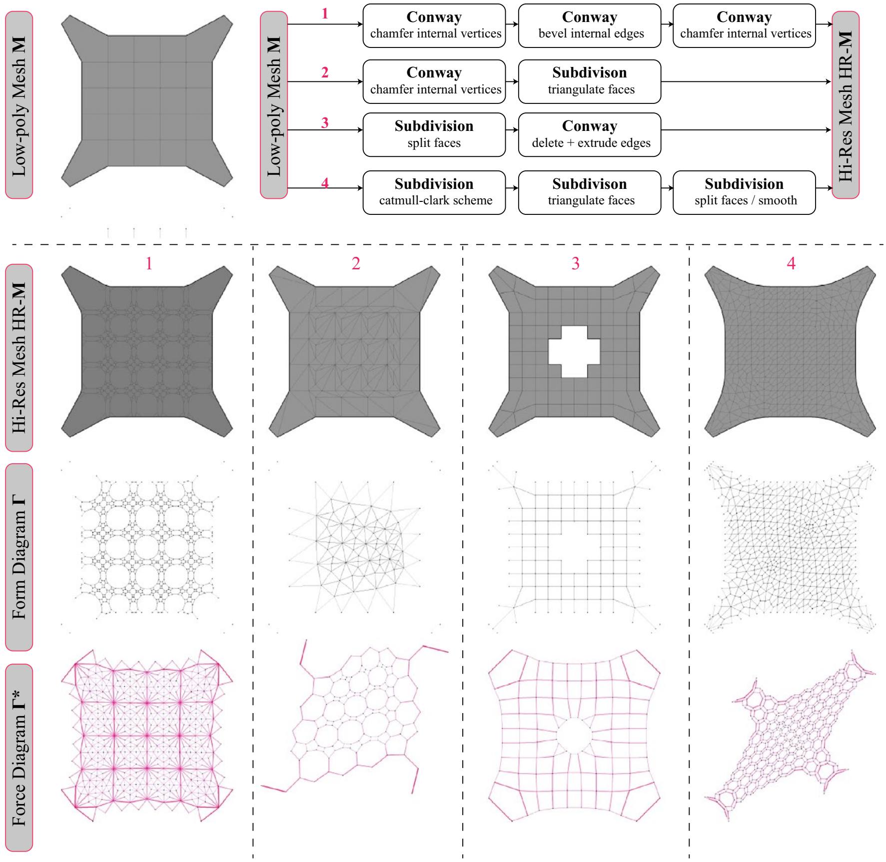

Fig. 5 Procedural generation of HR-M using Conway operators and subdivision scheme on M

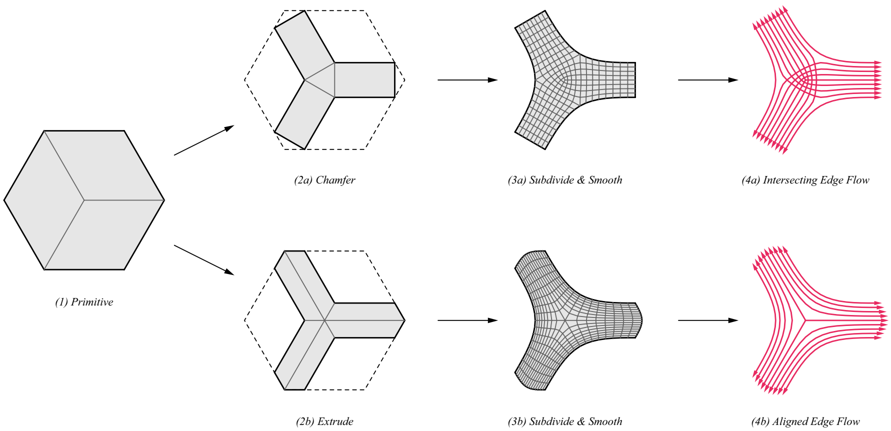

Fig. 6 Selection of structurally relevant Conway operator and subdivision schemes

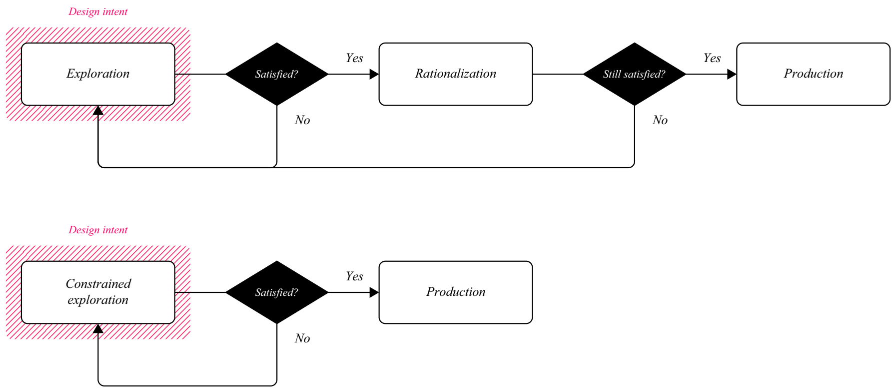

Fig. 7 Typical design production pipeline (top) and a collapsed design production pipeline (bottom)

5. Quick selection/picking tools. The picking of boundary vertices / edges as constraints in MME is reduced to constant time operations O(1) as compared to the current implementation in RhinoVault where it is proportional to the resolution of the input geometry (HR-M) i.e O(n).
6. Ease of subdividing a desired M using subdivision schemes such as Catmull–Clark subdivision scheme (Catmull and Clark 1978), modified Catmull–Clark subdivision scheme inbuilt in Autodesk® Maya (Stam 1998), Doo–Sabin scheme (Doo and Sabin 1978), mid-edge subdivision scheme (Peters and Reif 1997) etc. to create an higher resolution mesh HR-M. The advantages of using subdivision schemes to explore funicular shapes using 2D graphic statics have been established (Akbarzadeh et al. 2014) and we use such subdivision schemes in addition with Conway operators to procedurally generate HR-M from M, some of which are noted in Fig. 5. In the context of TNA and FDM, certain subdivision schemes are structurally more relevant as compared to others as highlighted in Fig. 6 (b being a better option than a).

Discussion

Impacts on Production

Exploration and rationalisation are typically treated as separate consecutive steps within contemporary design modelling workflows (Fig. 7). Design space is explored until an acceptable solution is found which is subsequently rationalised to comply with geometric constraints related to efficient production. If the rationalisation step results in unacceptable deviations from the previously accepted solution, then exploration is resumed in search of a different solution that better complies with the relevant production constraints. Otherwise, the design is moved into production. Given the prevailing use of mesh processing techniques for later stage design rationalisation tasks, embedding TNA within an MME presents a significant opportunity to collapse the traditional design to production pipeline in the context of funicular structures (Fig. 7). Geometric constraints related to efficient modes of production can be combined with those enforced by TNA, facilitating the interactive exploration of fabrication-aware funicular forms. In this context, intermediate steps between design and production are no longer required as the design model only exposes the space of solutions that can be efficiently fabricated and assembled. The design workflow developed during the AA Visiting School Chennai concisely demonstrates the impacts of a fabrication-aware form-finding process enabled by MME integration (Bhooshan et al. 2015a). The project sought to use of curved crease folded(CCF) formwork to cast nodal elements of a concrete funicular network structure. As detailed in Bhooshan et al. (2015b), the geometric constraints associated with CCF forms demanded a minimum amount of curvature at each node in the structure (Fig. 8). Enforcing these constraints within a modified FDM implementation enabled the rapid exploration of near-funicular solutions that were also foldable (Fig. 9)

Current Implementation of MayaVault

Our current implementation of TNA within the MME of Autodesk® Maya utilises the Maya application program interface(API) to augment the mesh creation and manipulation tool-set, dependency graph, brush-based tool-kits etc that exist within Maya, with a bespoke and persistent implementation of a half-edge mesh. Thus, our current implementation allows for the form-finding process to begin via

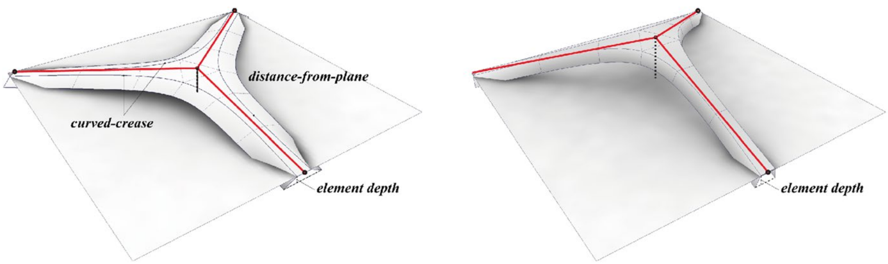

Fig. 8 Relationship between node curvature/depth and fold angle. Image taken from (Bhooshan et al. 2015c)

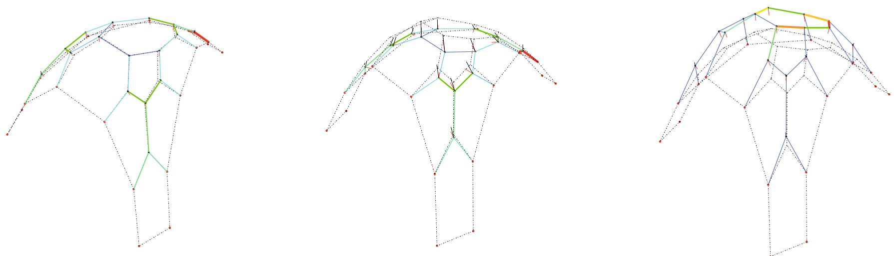

Fig. 9 Converging on foldable funicular network. Image taken from (Bhooshan et al. 2015c)

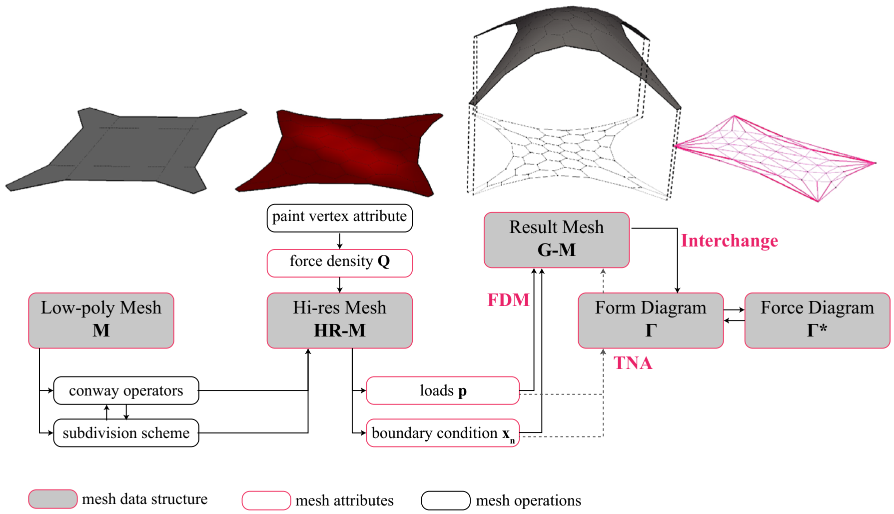

Fig. 10 Overview of workflow for FDM and TNA in a MME

semi-procedural creation of an initial topology as noted in section "Relevant Prior Work". Subsequently, an arbitrary 'painting' of force densities and the use of the FDM method establishes the necessary data for further inspection via TNA. Next, the form and force diagram can optionally be extracted from the unified mesh data-structure (Fig. 10). Lastly, the force densities derived from the TNA workflow can be algorithmically repainted back unto the mesh. We believe that this enables the designer to extend the currently popular edit-and-observe mesh design procedures with the didactic aspect of relating form and force diagrams that TNA affords.

Our current implementation includes most of the features specified previously (section "Introduction") apart from a constraint solver. Therefore an aspect of future work would be to include a robust constraint solver to address fabrication and/or spatially related constraints that may be discovered further downstream as the design pipeline progresses.

Conclusion

The current implementation of TNA in the publicly available RhinoVault and also within a partial, publicly unavailable MME Vouga et al. 2012; Tang et al. 2014, show that significant ground has been covered in exploring the foundational aspects of TNA—design thinking, mathematics, data structures, fast algorithms etc. This paper showed that incorporating TNA within an MME would improve the didactic value of the form-finding method. Further, the paper showed that such an incorporation would enable appropriating the skill of designers in creating and manipulating topology in the production of novel structural shape. The paper also specified and exemplified some of the features of the MME to enable the same. Further, most of the current research literature and implementations of TNA within an MME or without assume that the initial form diagram or mesh is generated arbitrarily by a user / architectural designer. However, this paper showed that most designers use combinations of a handful of Conway operators to produce the initial topology or form diagram. Thus unifying TNA within an MME could help the research community to leverage the procedural aspect of the generation of the form diagram, which is currently held tacitly within the design community.

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Vishu Bhooshan is a Senior Designer at Zaha Hadid Architects as part of the Computation and Design Research (ZH CODE) group at London. He completed his bachelor degree from Pune, India (2010) and his Master's degree at the Architectural Association, Design Research Lab (2013). He previously worked as an Architect in India at B.S Bhooshan & Associates, Mysore, and Mindspace Architects, Bangalore. His current interests are on 3D printing, digital form-finding and topology optimisation and statistical learning. He develops early design computational design tools for the above and in particular for data-driven design processes including gathering and visualisation of 3D urban city data. He has taught and presented/published work at various workshops, professional conferences and symposiums including CAADRIA '18, ACADIA '14 '17, simAUD '15 '17, IASS '16, AAG '16, eCAADe RIS '17, AAVS Global Summit '15, Hyper Threads AA Visiting School '13 - 17' and at various institutes in India, China and Brazil.

David Reeves is a Senior Researcher within the Computation and Design (ZH CODE) group at Zaha Hadid Architects where he leads the office's R&D efforts in geometry processing. He is also currently a design tutor within the March Architectural Design (AD) programme at the Bartlett School of Architecture. Previously, he has taught and presented work at various academic institutions including the Architectural Association, Aarhus School of Architecture, CITA, and Yale University. His work has also been published and presented at international conferences and events such as Acadia, simAUD, IASS and Smartgeometry.

Shajay Bhooshan s a PhD candidate at the Institute of Technology in Architecture, ETH Zurich, where he is a research assistant in the Block Research Group. He also heads the research activities of the Computation and Design Research (ZH CODE) group at Zaha Hadid Architects, London, having co-founded it in 2007. Previously, he worked at Populous, London, and completed his Master's Degree at the AA School of Architecture, London, in 2006. He has taught and presented work at various events and institutions across Europe, India, China, the USA, Brazil and Mexico. His current interests and responsibilities include overseeing design research and the development computational platforms for the same at Zaha Hadid Architects.

Philippe Block is Professor at the Institute of Technology in Architecture at ETH Zurich, where he directs the Block Research Group (BRG) together with Dr. Tom Van Mele. The BRG focuses on equilibrium analysis, computational form finding, optimisation and fabrication of curved surface structures, specialising in unreinforced masonry vaults and thin concrete shells. Prof. Block is also the director of the Swiss National Centre of Competence in Research (NCCR) Digital Fabrication. Within the NCCR, the BRG develops innovative structural design strategies using bespoke prefabrication, pushing construction innovation. Block studied architecture and structural engineering at the VUB in Belgium, and at MIT in the USA, where he earned his PhD in 2009. With the BRG and as partner of Ochsendorf DeJong & Block (ODB Engineering), he applies his research into practice on the structural assessment of historic monuments and the design and engineering of novel shell structures.