Compression Support Structures for Slabs

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Abstract. *This paper presents a novel and rigorous approach for the design of efficient spatial support structures for flat slabs. Based on the given dead load of the slab and the topology of the supporting structure, this method allows form-finding of support structures that are in equilibrium with axial compression forces only. During the form-finding process, the resulting horizontal forces from the compression support structure are balanced through translation and rotation of the slab, and subsequently resolved in the plane of the slab using funicular or trussed strut-and-tie systems. The presented method can be applied for the design of a variety of different typologies of supporting structures, such as inclined columns, curved walls, branching structures and shells. A computational prototype of the approach is implemented as CAD modeling tool, and the potential of the method is shown through two formal design explorations.*

Keywords: Structural Design, Form-Finding, Compression-Only Forces, Strut-And-Tie Systems, Computational Design Tools

Figure 1: Left: The *Basento Viaduct* in Potenza, Italy, by Sergio Musmeci, 1967-74. The flat bridge deck is supported by a concrete shell with complex geometry (Photo by nEmo Gruppo Architetti, Florence). Right: *Viscous Adaption* in Salzburg, Austria, by Soma Architecture, 2010-12. The flat, perforated roof of the foyer of the Building Academy is supported by asymmetric branching columns (Rendering by Soma Architecture, Vienna).

1   Introduction

The integration of structural constraints in early stages of the architectural design process with digital means has been topic of a variety of recent design and research projects [Oxman and Oxman, 2010]. A typology of structures that has been described only fragmentary in this context is the flat slab supported by compression-only structures. This typology offers a rich variety of formal expressions, since the support structure can take various forms such as inclined columns, branching structures, double-curved walls, or shell surfaces. Up to now, all of those solutions have been studied separately. The lack of a unified design approach is remarkable, as this system is structurally efficient and allows designing support structures with a minimum use of material. Furthermore, the typology has a wide range of architectural applications, since the slab can be used as roof, floor plate, or bridge deck.

Different approaches to the form-finding of such structures have been used in the past. The design of shell-supported slabs has been intensively studied by Musmeci using physical models [Nicoletti 1999]; based on these experiments the impressive *Basento Viaduct* in Potenza, Italy (Fig. 1, left) has been realized. For the form-finding of branching structures, physical experiments as well as computational means have been applied [Otto and Bien, 1992], [Hunt et al. 2009]. Sasaki developed the *3D Extended ESO Method*; it combines FEA software with shape optimization. This technique has been used in the design of the branching structure for the *National Convention Centre*, in Doha, Qatar [Sasaki 2007].

Several buildings have been constructed, that incorporate flat slabs supported by non-standard structures, working primarily in compression: *Terminal 1* of the Stuttgart Airport, Germany, and the *Pragsattel II* footbridge [Holgate 1997]; both are branching structures designed by Schlaich, Bergermann & Partner. Furthermore the *Mediatheque* in Sendai, Japan, designed by Ito and Sasaki, consists of thin floor plates that are supported by a network of steel tubes forming spatial, hyperboloidal columns [Rappaport, 2007]. The perforated roof of the new foyer of the Building Academy Salzburg, supported by asymmetric branching columns, was recently completed (Fig. 1, right) [Oberascher 2012].

This paper describes a unified design methodology for compression-only support structures for flat slabs. It is structured as follows: Section 2 states the general problem and frames it within the concept of lower-bound design. Section 3 describes the design approach and form-finding algorithm in two steps, and illustrates it with an implemented prototype. Section 4 finally presents two formal design explorations for supporting structures with complex geometry.

2   Concepts

The approach is suited for early-stage design explorations of spatial compression-only support structures, based on one dominant loading case, for example, the self-weight of the slab. The method is considering static equilibrium only, and allows making constructive design decisions based on qualitative and quantitative properties of the resulting force-flow. In subsequent steps, a structural analysis

regarding structural safety for live loads, stability (e.g. buckling) and serviceability has to be carried out, and if necessary, additional design features for stiffening have to be introduced.

2.1   Lower-Bound Solutions

Within the framework of the theory of plasticity, and thus assuming ductile behavior of the building materials, the transfer of loads can be described by only considering static equilibrium of forces while neglecting material stiffness and deflections. All structures discussed here are modeled as pin-jointed structures with only axial forces in their members. According to the lower-bound theorem of the theory of plasticity, such structures are safe for a given loading case, if the member dimensions for a given state of equilibrium are sufficient for axial stresses and are not in the danger of buckling. Lower-bound solutions have been successfully used in the design of steel structures [Baker et al. 1956], structures in reinforced concrete, using *strut-and-tie models* [Schlaich et al. 1987] and *stress fields* [Muttoni et al. 1996], and masonry structures, using *thrust lines* [Heyman 1995] and *thrust networks* [Block and Ochsendorf 2007]. Recently, lower-bound concepts have been combined with parametric modeling methods for the design of roof and bridge structures with complex geometry [Lachauer and Kotnik, 2010], [Lachauer and Kotnik, 2011].

2.2   Equilibrium of Supported Slabs

The main challenges in finding an equilibrium solution for a support structure with complex geometry for a flat and heavy slab are illustrated schematically for a simple two-dimensional case with a pin-jointed branching column (Fig. 2). The self-weight of the support structure is not considered, as it is assumed to be small in comparison to the weight of the slab.

Assuming that the support positions are given, one can determine a set of support forces F_1^v, F_2^v for the dead load Q of the slab (Fig. 2a). For the branching support structure with arbitrary geometry, shown in Figure 2b, the forces in the strut elements connected to the slab are statically defined: using trigonometry, the axial member forces, F_i, and horizontal components in them, which are the horizontal support forces, F_i^h, can be directly found from the vertical support forces, F_i^v, as indeed $F_i = F_i^v + F_i^h$. Considering the free[1] node N, the resultant, or sum of forces in the struts attached to the slab, $F^r = F_1 + F_2$, is not necessarily acting in the direction of the strut connected to the ground, which means that node N is not in equilibrium. Furthermore, the reaction force $-F^h = F_1^h + F_2^h$, acting horizontally in the plate, is also not necessarily balanced yet.

With these observations as premise, the form finding problem can thus be divided in two categories:
- Finding the geometry of the supporting structure of *horizontally restrained* slabs (e.g. a projecting roof attached to a building), such that all of its free nodes are in equilibrium for the given loads of the slab. This is theoretically

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[1] A *free* node is neither a ground support, nor a support of the slab.

possible for any given support position G to the ground, by just moving the free nodes N, because the remaining horizontal reaction force F^h can be taken by the horizontal restraint of the slab. (Fig. 2c).

- Finding the geometry of the supporting structure of *horizontally unrestrained* slabs (e.g. a roof of free standing pavilion), such that all of its free nodes are in equilibrium for the given loads of the slab *and* the remaining horizontal force components in the plane of the slab are equilibrated. Therefore, both the positions of free nodes *and* ground supports have to be modified (Fig. 2d).
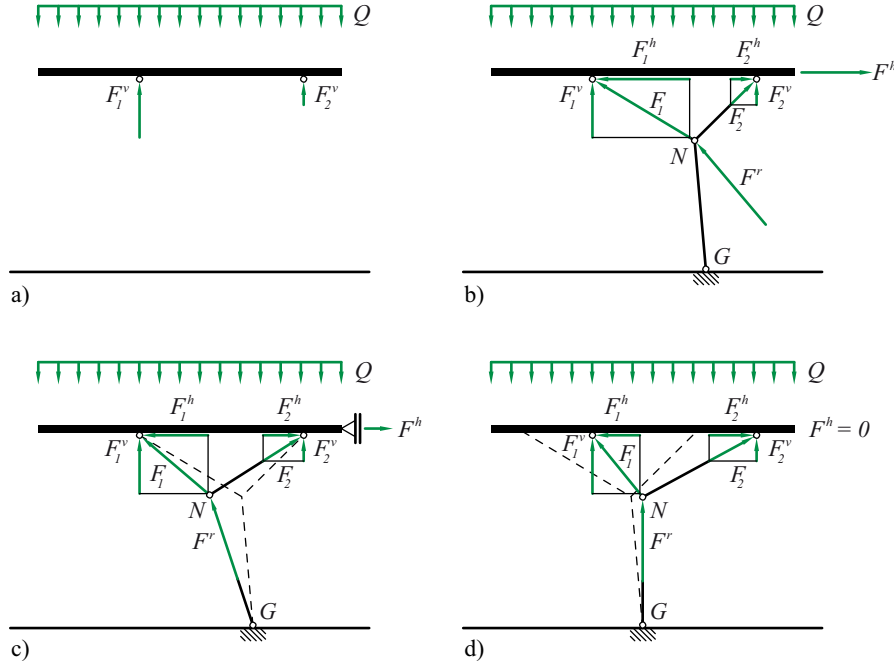

Figure 2: Given a) dead load Q of the slab and support forces, F^v, b) for an arbitrary geometry, the support structure in not in equilibrium in node N, as forces F^r and F^h are not balanced; c) shows a support structure in equilibrium with a horizontally restrained slab; and d) a support structure in equilibrium with the slab without horizontal restraint.

3   Method

In this section the form-finding method will be described for the two categories, horizontally restrained and unrestrained slabs. In Section 3.1, the generation of support structures that balance the vertical force components in the slab, is described. This method can be used for designing horizontally restrained slabs. In Section 3.2, additional equilibrium conditions are formulated as extension to the method described in 3.1, in order to solve for both vertical and horizontal force components simultaneously. This method can be used for designing horizontally unrestrained slabs.

The form-finding method is applied to a slab with given self-weight and support positions. Furthermore, a set of vertical support forces, $F_1^v \ldots F_n^v$, balancing the dead load Q of the slab is assumed as given[2] (Fig. 3). Additionally, a network of struts in space is given. These determine the connectivity of the structure and the starting point of the iterative form-finding procedure. The only restriction to the connectivity of the network is that each support of the slab has to be connected to exactly one strut, as otherwise the member forces of these struts cannot be uniquely defined; the nodes at the supports of the slab would become statically indeterminate.
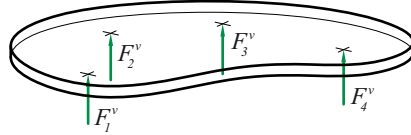

Figure 3: A generic plate with given support positions and forces.

3.1 Finding Vertical Equilibrium

As described in Section 2.2, for a horizontally restrained slab (Fig. 2c), the objective is to find the position of the free nodes N_i such that those nodes are in equilibrium. If the slab is restrained in at least two points, the resulting horizontal force components F_i^h in the slab can be equilibrated within the plane of the slab either with a tension or compression funicular or a by a truss.

Two conditions have to be satisfied for the vertical equilibrium of the structure: For each support i of the slab, a horizontal force F_i^h has to exist such that the support force F_i^v can be balanced by the force F_i in the supporting strut:

$$F_i = F_i^v + F_i^h \qquad 1)$$

For each free node N_i, the forces S_j in the m neighboring struts have to be in equilibrium, so the residual force R_i at each node has to be zero:

$$R_i = \sum_{i=1}^{m} S_i = 0 \qquad 2)$$

In the two-dimensional example (Fig. 2c and 2d), for node N, the forces in the neighboring struts would be $S_1 = F_1$, $S_2 = F_2$, and $S_3 = F^r$. In order to achieve the two equilibrium conditions 1) and 2), the structure is solved as a tension network consisting of zero length springs [Harding and Shepherd 2011], and subsequently the sign of the forces is switched, resulting in a compression-only solution.

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[2] Note that again within the lower-bound theory, these can be chosen, if it is indeed assumed that the slab has enough bending stiffness to distribute the forces in those proportions to the supports. One possible set of support forces could be obtained using an FE analysis tool.

Starting with the provided initial geometry of the network of struts and the vertical force components $F_1{}^v \dots F_n{}^v$, the initial forces $F_1 \dots F_n$ in the struts are computed using trigonometry (as described below in step I). Next, the scalar c is calculated, defined as the inverse of the average magnitude of the initial forces $F_1 \dots F_n$ in the struts connected to the slab. This scalar relates the level of pre-stress of the springs to the magnitude of given vertical force components.

In each step of the form-finding process, the following tasks are performed:

I. The forces in the struts connected to the slab are calculated as

$|F_i| = c \cdot |F_i{}^v| \sin^{-1} \alpha_i$, whereby Eq. 1) is directly satisfied, α_i being the angle between strut i and the slab.

II. All struts that are not connected to the slab supports are modeled as zero length springs; the forces S_i are proportional to the strut length, with an initial level of pre-stress of 1: $S_i = l_i / l_i^0$; l_i is the actual length of the strut, and l_i^0 is the initial length of the strut.

III. The position of each free node is updated: $N = N + k \cdot R_i$, k is a small scalar, defining the step size of the procedure (e.g. $k = 0.1$).

This procedure is iteratively repeated, until the sum of residual forces $\sum R_i$ is smaller than a given threshold ε. After convergence, all forces have to be divided by c in order to calculate the real member stresses and support forces that are in equilibrium with the given vertical loads $F_1{}^v \dots F_n{}^v$. In order to accelerate convergence, one might introduce velocities to solve the problem with a *dynamic relaxation* formulation [Barnes 1999] or apply more advanced solving strategies, such as a *Runge-Kutta* solver [Kilian and Ochsendorf 2005]. Instabilities in the solving procedure occur due to vanishing spring lengths in the case of very large differences in the magnitudes of the given vertical forces.

3.2 Finding Horizontal Equilibrium

In order to find an equilibrium state of the support structure, with the additional constraint that all horizontal force components in the plate should be in equilibrium, hence resulting in a free-standing structure, two more equilibrium conditions are added to Equations 1) and 2). A set of in-plane forces $F_i{}^h$ is in equilibrium if and only if their sum is zero:

$$R = \sum_{i=1}^{n} F_i{}^h = 0 \qquad \qquad 3)$$

and if the sum of the moments they induce, around any point in the plate, is zero:

$$M = \sum_{i=1}^{n} M_i = \sum_{i=1}^{n} F_i{}^h h_i = 0 \qquad \qquad 4)$$

with h_i being the perpendicular distances to the line of action of the force F_i^h from a chosen point O (Fig. 4).

To integrate these two additional constraints in the form finding process, the slab is translated and rotated, inclusive its supports, at each iteration, such that its position and rotation moves towards a balanced state. Therefore, one should initially define an arbitrary point O to calculate the resulting moment M.

In each step of the solving procedure described in Section 3.1, two more steps are added, which are performed subsequently after step III:

VI. The system of the slab, together with its supports and point O, is rotated in plane by the angle $k_1 \cdot M$ degrees.

V. The system of the slab, with slab supports and point O, is translated horizontally by the vector $k_2 \cdot R$.

Both k_1, k_2 are a small constants defining the step size of the solving procedure. Experience has shown that k_1, k_2 being about two order of magnitude smaller than k renders good results.
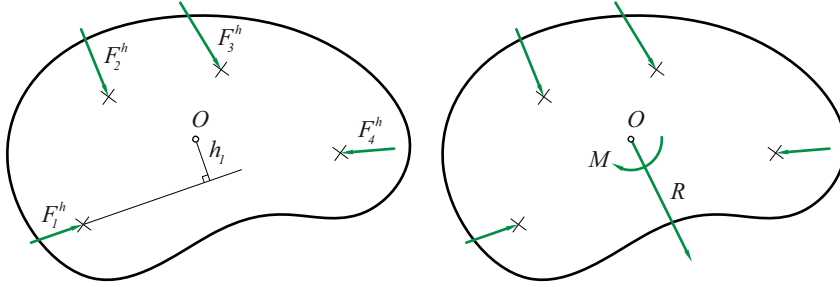

Figure 4: left: slab with horizontal force components $F_1^h \dots F_n^h$ acting at the supports, and the perpendicular distances h_1 from the chosen point O to their line of action; right: slab with the equivalent actions M and R.

3.3   Implemented Prototype

Based on the algorithm described in Section 3.1 and 3.2, a prototypical tool for form-finding efficient support structures for slabs has been implemented for the CAD software Rhinoceros [McNeel 2012], using the scripting language RhinoPython [Baer 2012]. A dynamic relaxation method is implemented as solver for the implementation.

As input, the user has to draw a starting geometry for the support structure as a network of lines, define slab and ground supports as points, construct the outline of the slab as a closed curve, and provide the loads of the slabs as vertical lines, drawn from the slab supports. The tool has two modes: "horizontally restrained", in which case the position of the slab is kept fix (Fig. 5a-b), and "horizontally unrestrained", in which case the slab, together with its support points, are translated and rotated until a position is found for which the resulting rotational

moment and force resultant from the horizontal force components in the slab vanish (Fig. 5c-d).
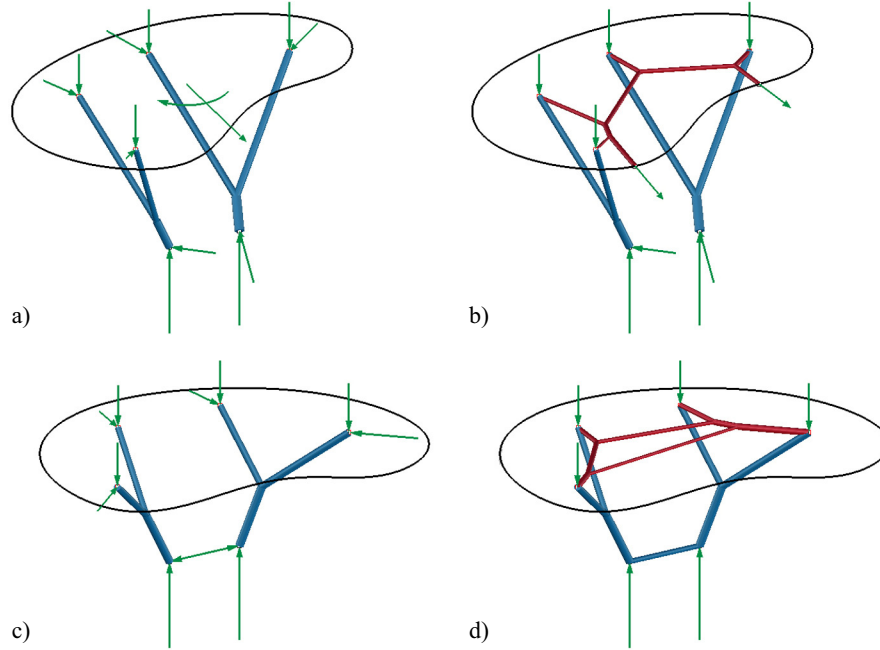

Figure 5: a) support structure for a slab that is horizontally not in equilibrium, a resulting force and a moment are acting; b) a funicular system, constructed within the plane of the slab, transferring the horizontal support force components to the slab's external restraints; c) the slab has been rotated and translated, the slab is horizontally in equilibrium; d) a funicular tension system balancing the horizontal force components in the slab, and a horizontal compression strut balancing the horizontal force components at the ground supports.

As soon as the algorithm converges and a state of equilibrium is reached, the forces in the struts are visualized as cylinders that have a section area proportional to its internal stress. Furthermore, the horizontal and vertical reaction forces of the support structure is generated as lines with arrowhead, both at the slab supports and at the ground supports (Fig. 5a-d). Depending on the layout of the horizontal forces, it might be possible to elegantly balance the horizontal reactions using a tension funicular restraint to the external restraints (Fig. 5b), or using a tension ring (Fig. 5d). In more complex cases it might not be possible to find a funicular solution, but it is always possible to use an in-plane strut-and-tie system, e.g. a truss, instead. Funicular solutions can be generated using techniques from graphic statics [Allen and Zalewski 2010], possibly implemented as computational modeling tool [Lachauer et al. 2011].

4   Design Explorations

To demonstrate the power of the presented approach, two formal design explorations of free-standing, horizontally unrestraint roofs are presented. In the first case (Figs. 6 and 7), a branching structure with complex geometry is supporting a slender slab in a regular grid, while the ground supports are placed freely, reacting to potential special site constraints. Input network and resulting geometry are shown below (Fig. 6). In the second case (Fig. 8), a thick, cantilevering slab is supported by inclined, double-curved quadrilateral networks.
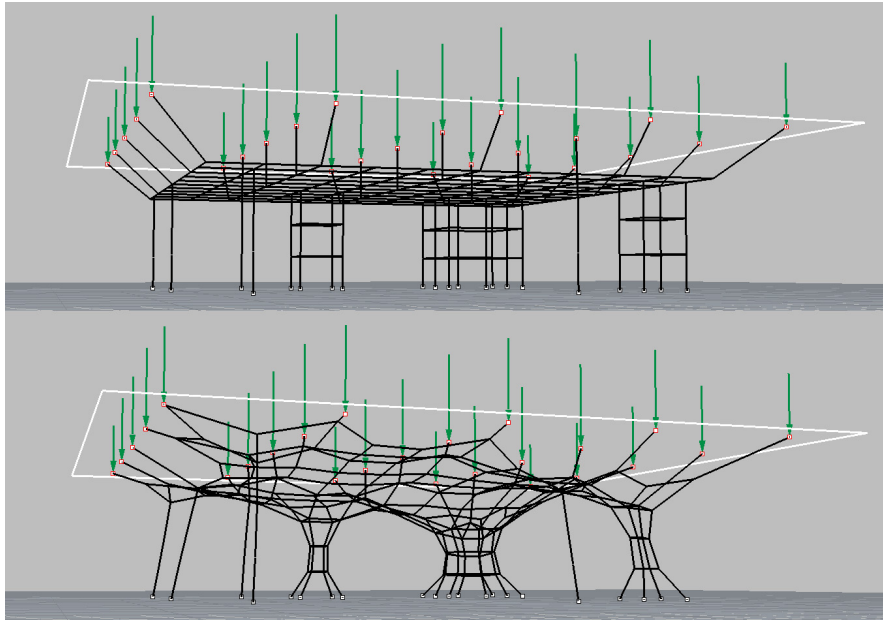

Figure 6: above: input geometry with support network (black lines), roof outline (white rectangle), dead load forces (green arrows), and roof respectively ground supports (red and black points); below: resulting compression-only support structure, the roof is minimally rotated, and the slab is globally in balance.

The shown supporting structure in both cases is the result of the direct translation of the force network into a tubular frame structure. To withstand horizontal and asymmetric load cases, it is assumed that the support structure acts in bending in those load cases, hence omitting the need for a stiffening scheme, such as with diagonal struts. Instead of literally materializing the network of struts, one could also interpret the resulting network as a three-dimensional discretization of force paths within surface structures such as shells or doubly curved walls. Especially novel fabrication techniques for surfaces structures such as robotically fabricated brick walls [Bonwetsch et al. 2007] and fabric formwork for concrete shells [Van Mele and Block 2011] may be paired are with this design method.
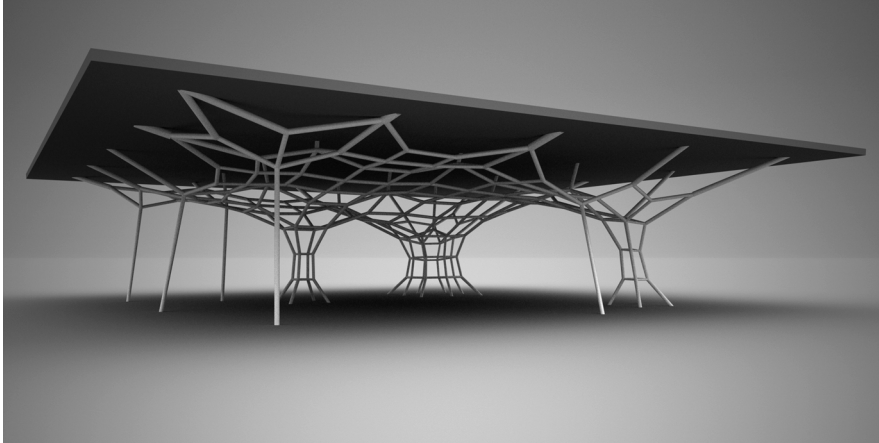
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Figure 7: A slender concrete slab with the dimensions of 35 x 35 m, supported by a compression-only branching structure made out of steel. The free-form support structure forms a regular grid of 5 x 5 supports, with an 8 m spacing and is horizontally unrestraint.
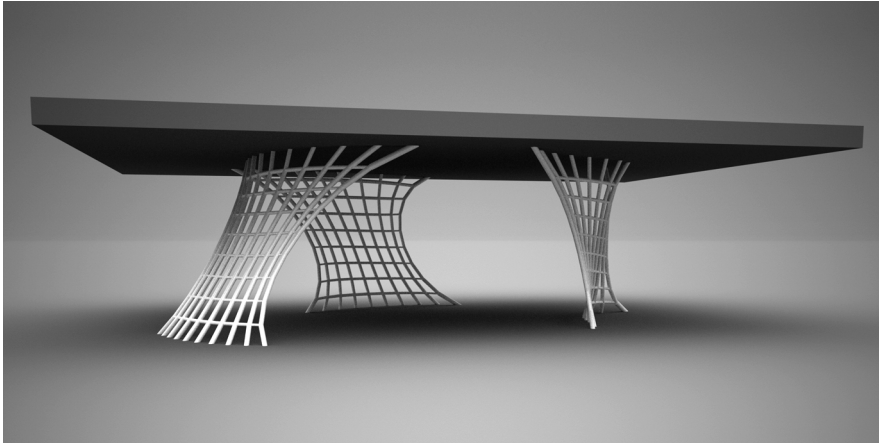

Figure 8: A thick, cantilevering, and horizontally unrestraint slab with the dimensions of 10 x 14 m, supported by three doubly-curved, wall-like networks.

5   Discussion

The presented method is highly general and extensible, but still the solving procedure of the implemented tool can be improved. The presented examples solved within a few seconds on an Intel Core Duo Processor with 2.8 GHz. Solving speed could easily be improved multiple orders of magnitude by using a more advanced solver [Veenendaal and Block 2011], but also by using a compiled plug-in instead of a prototype implementation written in a scripting language. Currently the only way to influence the result of the form-finding process is to

vary the input geometry and topology of the supporting network. Especially when dealing with support forces that differ strongly in magnitude, the resulting networks may be distorted or instabilities might occur during the solving process. A direct control of the initial level of pre-stress of the springs by the designer would provide additional design freedom that is inherent to such complex, mostly structural indeterminate structures.

6 Conclusion and Future Work

The presented method offers a formally flexible and technically simple algorithm to design compression support structures for slabs, allowing the exploration of interesting hybrid structures combining e.g. branching and surface structures. It provides insights in the global equilibrium of the supported-slab system, not only of the support structure, but also of the in-plane forces of the slab. Future work will focus on the enhancement of user control, especially the initial level of pre-stress of the springs. A possibility for further generalization is the acceptance of non-vertical support forces as input, thereby enabling the "stacking" of such non-standard compression supported slab systems to multi story buildings.

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