Digital Stereotomy: Voussoir geometry for freeform masonry-like vaults informed by structural and fabrication constraints

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Summary

This paper discusses new ways of digitally generating voussoir geometry for freeform masonry-like vaults. Through the combination of the recently developed form finding methods like Thrust Network Analysis (TNA), which facilitates the design of freeform compression-only surfaces and fast and flexible CNC-machining, structural efficient and expressive stone structures could be build which embrace today’s technological, economical, and ecological demands. This can only happen when all constraints in the process of design and materialization are fully understood and integrated in a smooth digital stream from form finding to fabrication. This paper focuses on the geometrical interdependencies between the generation of the voussoirs and the physical limitations of the fabrication on the basis of structural information. The principle is tested physically by cutting individual foam blocks, simulating the rapid and efficient cutting of natural stone with a diamond-wire saw. This is done on an experimental fabrication setup, especially designed for this research and based on four-axes CNC hot-wire cutting technology.

Keywords: *Digital stereotomy; stone cutting; formfinding; digital fabrication; CNC wire-cutting technology; freeform masonry vault design; voussoir geometry;*

1. Introduction

Gothic masonry vaults show in an impressive manner the natural aesthetics of stone structures in which structural and ornamental building parts are combined to complex building forms. To master the geometrical complexity and realization of these forms, the art of stone cutting (stereotomy) emerged and developed over centuries [1].

To design and realize those delicate structures, stereotomy is mainly planned and developed by using layout drawings, the so called ”traints”, which show the orthographic projection of typically complex and expressive stone structures [1]. Historic masonry structures mostly consist of symmetric and/or repetitive building parts though. This geometrical characteristic facilitated the tessellation design of the vault and the shape of the voussoirs, which could be determined based on experience and basic assumptions on the direction of forces along arches and ribs. With the advent of the industrial revolution and the rise of new structural materials and systems, this technique fell into oblivion, which caused a major shift in architecture, leaving stereotomy stagnating until today [2].

Equipped with new digital design and fabrication tools, we are now encouraged to rediscover the great potential of building elegant and efficient stone structures. Thanks to new form finding methods [3,4], it is possible to design freeform, compression-only vaults in a digital CAD environment in structurally intuitive ways [5]. On the fabrication side, CNC machines designed to process stone efficiently are already widely used and well established in the industry [7,8]. Computational approaches have been used to generate and visualize complex three-dimensional representations of stone vaults. These approaches incorporate the analysis and investigation of existing structures but also the modification and manipulation of certain tessellation patterns on double curved geometries by using three-dimensional modelling software and build-in transformation commands [9].

These developments and technical advances clearly hint that there could be a future for new types of stone vaults. This mainly embraces the use of new form finding methods, especially e.g. TNA, which significantly increase the flexibility of designing and shaping compression-only vaults, resulting in a wider spectrum of vault designs in general. In addition, the automated process of CNC stone cutting enhances the slow and meticulous procedure of cutting complex elements.

However, with the emergence of asymmetrical, double-curved forms new and complex challenges arise concerning the spatial generation of appropriate tessellations and the geometry of individual voussoirs. In that respect, two-dimensional, orthographic projections seem to have little relevance for the future of advanced vault design, with inherently non-repetitive, three-dimensional building parts. A particular challenge is the generation of vault tessellation and voussoir geometry, based not only on architectural and aesthetical considerations, but also integrating structural information as well as fabrication and assembly constraints within an informed and cross-linked computational framework.

Except for the new challenges related to “geometrical” complexity, the building industry is also facing environmental constraints. “Freeform” is no longer accepted at any cost; efficiency of material use is a key consideration to embrace today’s economical and ecological demands.

With respect to the above mentioned aspects, it is clear that the state-of-the-art of stereotomy research cannot provide a satisfactory method to generate and process the individual voussoirs and their continuous bond in freeform masonry-like vaults. In the following sections of the paper, we will introduce the main components of the above mentioned framework, focussing on the aspect pertaining informed voussoir geometry. Section 2 will introduce the interdependent constraints with respect to the structural behaviour and the process of materialization. In Section 3, the fundamentals and strategies of informed voussoir geometry will be discussed. Sections 4 will show details of an experimental fabrication setup, developed to directly test the proposed approaches through scale prototypes. Finally, conclusions of this study are given in Section 5.

2. Interdependent Geometrical Constraints

The interdependent constraints concerning the generation and arrangement of voussoirs derive from three fundamentals of freeform vault design (Fig. 1):

1. the desired shape of the vault and its tessellation or pattern is subject to specific architectural and tectonic requirements and design intents;

2. the structural behaviour of the vault, including the local distribution and direction of forces, specify not only the structural shape, but also the stereotomic design; and

3. fabrication and assembly constraints limit the materialization of the masonry-like vaults.

2.1 Architectural and Tectonic Requirements

From all aspects which influence the overall shape of the vault by contextual, functional and visual considerations, the architectural and tectonic requirements and design intents are the softer ones. Moreover, specific patterns can be used as guidelines for the configuration of individual elements of the structure [9,2].

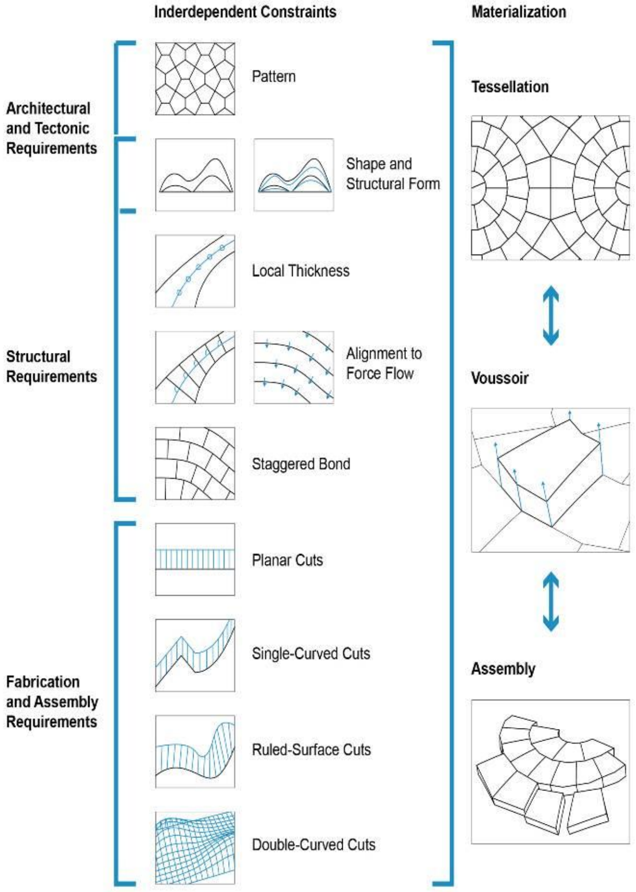

Fig. 1: Interdepending constraints

To find a structural shape, which is needed for a compression-only masonry vault, the surface is designed by TNA form-finding methods from scratch [3,4] or by automatic approximation of a given surface [6], combining good structural form with the architects' design intent. This surface is represented by a continuous NURBS-surface for subsequent, digital processes. To define certain, visual guidelines for the tessellation of the surface, initial topologies and rules can be defined using network definitions (e.g. using a branch-node matrix) for further implementation.

2.2 Structural Requirements

Structural requirements determine the local thickness of the vault, and hence also the local extrusion values for the voussoir generation. Additionally, the orientation of the tessellation, and therefore the contact faces of neighbouring voussoirs, wants to be aligned to the local force flow. By controlling minimal and maximal overlaps a staggered bond of the voussoirs in the structure is ensured, providing the necessary interlocking between the discrete elements to avoid sliding failure. The structural information of the compression-only vault is represented by a set of local surface information about the magnitude and direction of forces within the thrust network. This information is carried out by using TNA and stored in reference to the local coordinates (*u*,*v*-parameters) on the NURBS-surface which defines the overall geometry of the structure [3,4]. The spatial tessellation of the surface is then iteratively generated by a surface-constrained relaxation which results in an optimal configuration with respect to the above defined requirements (cf. Section 2). A weighted and hierarchical approach will be implemented in future research to embrace the conflicts between interdepending parameters and to generate possible solutions under different criteria.

2.3 Fabrication and Assembly Requirements

The fabrication and assembly requirements determine geometrical constraints which will inform the continuative materialization process. These constraints are defined by possible fabrication techniques used in automated stone machining. Circular saw machining usually result in planar surfaces, where extrusion cuts with two-axes wire stone saws are described by single curved surfaces. More geometrical flexibility is accomplishable with three-to four-axes wire stone cutting technology capable of cutting ruled surface geometries. In terms of geometrical flexibility, this is being seconded only by automated freeform stone milling, for instance by five-axes robotic stone cutting technology. The maximum and minimum dimension and volume of the voussoirs is defined by the physical machine setup used for fabrication and assembly.

These geometrical limitations and the machine setup need to be specified in a bidirectional way, balancing the technical feasibility of the machine process and the geometrical flexibility needed. For instance, five-axes robotic milling would allow for a maximum of geometrical flexibility for the generation of voussoirs (double curved faces), whereas straight cuts with a circular saw would limit the geometrical flexibility the most (planar faces). On the other hand, the efficiency in terms of fabrication time, material waste and availability of these technologies needs to be taken into account (cf. Section 3.1).

2.4 Materialization Process

The materialization process of masonry-like vaults itself is geometrically defined in three phases, specifically: the generation of a tessellation, the description of the individual geometry of the voussoirs and the construction sequence respectively the assembly strategy of the structure. All phases are based on the above mentioned geometrical constraints.

These listed geometrical constraints cannot be integrated as discrete, autonomous parameters for the further process of materialization. In contrast, most constraints are highly interdependent, demanding a weighted multi-variable optimization strategy aiming for a ”best fit” configuration of the tessellation and the voussoir geometry, considering different criteria like structural stability, efficiency of material use, smoothness of the approximated shape, effectiveness of the construction sequence, etc. Subsequently, structural models and discrete element methods have to be used to evaluate various solutions.

3. Informed Voussoir Geometry

The previous section of the paper gave an overview of all the geometrical constraints, informed by design, structure and fabrication, to be considered for the tessellation of a freeform vault and the generation of its voussoirs. This section focuses in particular on the influence of these geometrical constraints on fabrication techniques and vice versa, coupled and informed by structural information.

3.1 Geometry and Fabrication

Of all subtractive processes to produce discrete, solid block elements, wire stone cutting is of particular interest for the objectives set out above. Most importantly, it provides a certain degree of geometrical flexibility, which is needed for freeform vault design, thanks to the complex ruled-surface cuts possible with a 4-axes wire saw. In addition, wire stone cutting is the preferable technique considering the production requirements according to precision, speed and clearance [8].

Another advantage is the efficient use of material. In contrast to e.g. milling the cut-off material remains as a block, which can be used for smaller scale masonry.

3.2 Voussoir Generation

Ideally, the force flow is perpendicular to the main load bearing faces of neighbouring voussoirs. Therefore, the interface surfaces should be normal to the thrust surface. Working with extrusions and lofts, we identify that all those faces can be ruled surfaces. This can be explained by comparing the generation of the voussoirs and corresponding distribution of forces of a barrel vault, an axially symmetrical dome structure, and a double-curved freeform vault (Fig. 2).

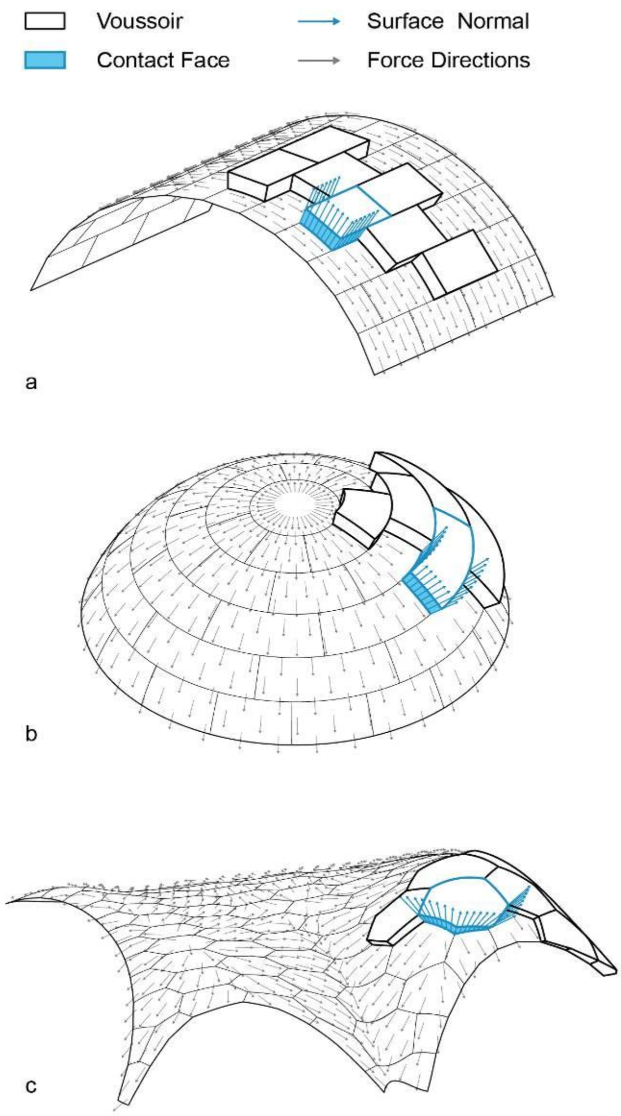

Fig. 2: ruled surface contact faces

One basic principal of the force flow of a compression-only structure is the orientation of the forces along the thrust surface. The normal of any contact face of the voussoirs needs to be tangent to the thrust surface. This needs to be guaranteed, in order to prevent instability through sliding between the voussoirs. In other words, a contact surface is described by lofting through a set of lines normal to the thrust surface which is by definition a ruled suface.

Fig. 2.a shows a barrel vault with a segmented tessellation resulting in planar contact faces. The dome in Fig. 2.b is smoothly tessellated along the longitude of the dome, which generates single curved contact faces. The freeform vault tessellation shown in Fig. 2.c has twisted ruled surfaces. All contact faces of these examples are ruled surfaces, and are thus theoretically possible to process with wire-cutting machines. Other constraints, such as clearance and angle restrictions of the CNC machine and self-intersection of cuts during the cutting process need to be checked.

Depending on the curvature of the vault, the intrados and extrados of the voussoirs can be globally or locally approximated by ruled surfaces [10,11]. In case this approximation is insufficient for strong curvature geometry, the fabrication technique can be combined with multi-axes milling to locally process double-curved, freeform geometry [7].

4. Experimental Setup for Automated Fabrication

4.1 Experimental, Physical Setup for Automated Fabrication

The results of the method described in the previous section were generated and physically produced on a customized 4-axes CNC wire cutter. The device was designed and constructed considering the geometrical flexibility needed for cutting individual voussoirs of freeform vaults. The experimental fabrication setup designed for this research, is based on hot-wire cutting technology. It allows the exploration of the geometrical interdependencies between the form of the voussoirs and the physical limitations of the machine. The individual elements are cut out of foam blocks, basically simulating almost one-to-one the rapid and efficient cutting of natural stone with a diamond-wire saw [8]. In Fig. 3 illustrates its geometrical configuration based on four axes. Besides a linear axis (X = 980 mm) for the frame movement, it comes with two individual axes (U,V = 910 mm) to guide the wire, which are attached along the vertical parts of the frame. A turntable as the forth axis (A = 360°) is fully integrated in the machine process and provides 360 degrees of cutting clearance.

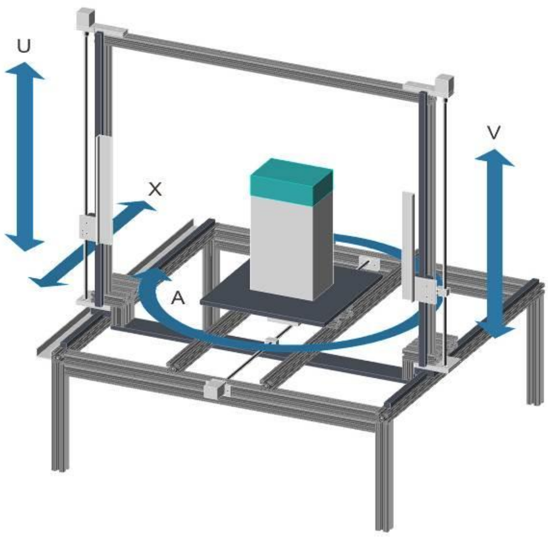

Fig. 3: Customized 4-axes CNC wire cutter

4.2 Software Development for 4-Axes CNC wire cutting

The developed CAM technology comprises an in-house written (by the first author) program to generate the necessary tool path data for the machining of solid foam blocks.

Fig. 4 describes the software routine to process the geometrical objects by analysing, detecting and exporting the specific data and information [12,13,14].

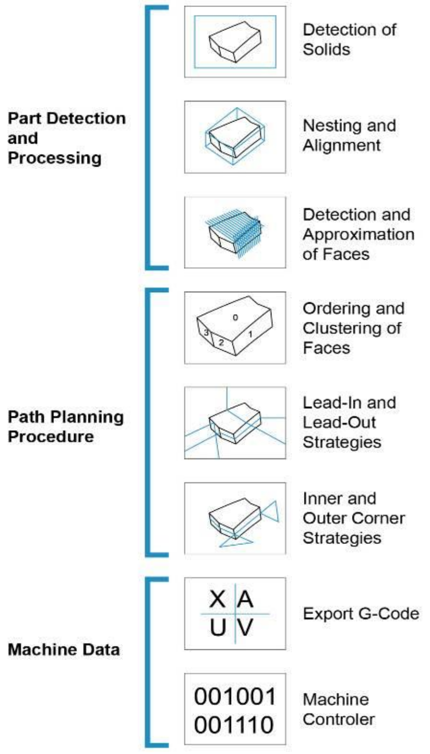

Fig. 4: Program sequence of developed software

The first step of the fully automated routine is to define the individual solid (NURBS polysurface) and the dimension of the initial material block (bounding box). The shape will then be aligned for the "best fit" position within the material block, in order to reduce the cut off volume and hence save material. Next, the individual surfaces are analysed and approximated within given tolerances (for non-ruled surfaces) or aligned (for doubly ruled surfaces) to fit the machine setup best. Furthermore, plausibility checks are made to detect possible self-intersection cuts of surfaces or violations of defined angle restrictions by the machine setup. The surfaces are then getting clustered and ordered in a way that the last bottom cut will finally release the voussoir from the material block. Specific lead-in and lead-out strategies as well as tool path optimisation for concave and convex creases are used to export the final machine code. This G-Code gets generated as a standard CNC program (NIST

RS274NGC) and fed to the machine controller via USB [15]. The above described program is written in the programming language Visual Basic and integrated as a plug-in in the CAD software Rhinoceros 4.0 [16].

4.3 First results

Fig. 5 shows the cutting process of a single voussoir undergoing different stages. The Styrofoam block is mounted on the turntable, while the hot wire is travelling through the material. This voussoir sample is one single piece of a patch of highly individual part geometries, as shown in Fig. 6. This model serves as a proof of concept for the efficient realisation of voussoirs with complex geometries for freeform masonry-like vaults.

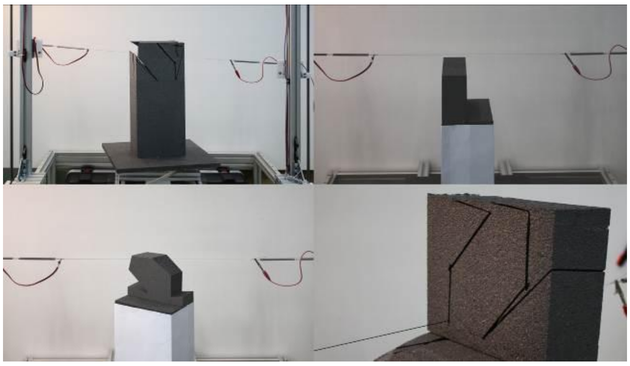

Fig. 5: 4-axes CNC wire-cutting in progress

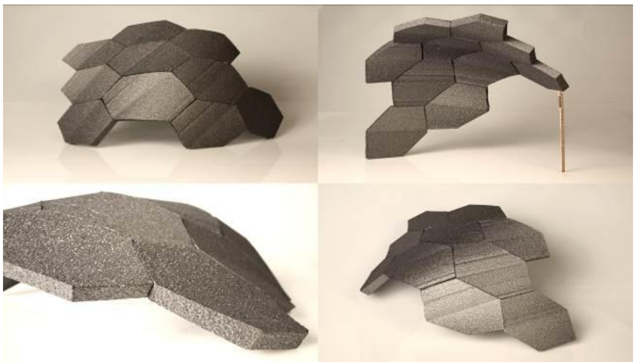

Fig. 6: Staggered voussoir foam samples

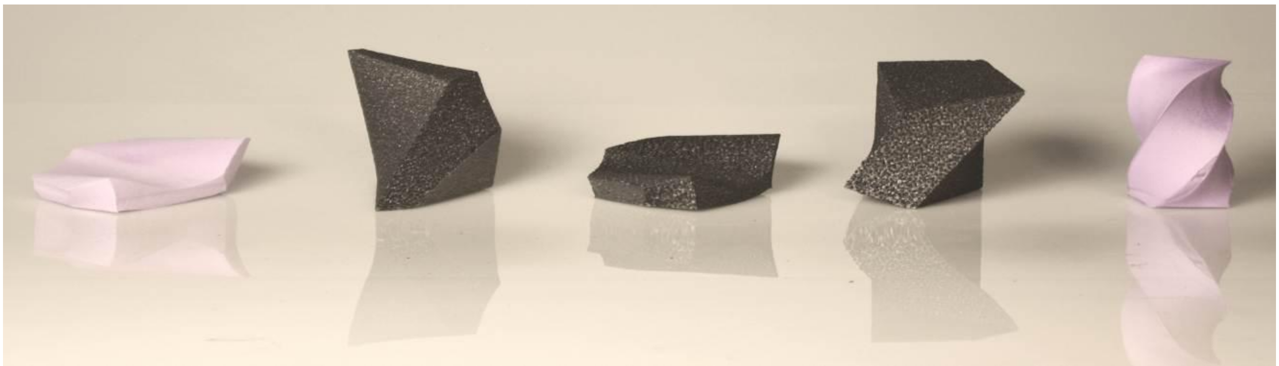

Fig. 8: Sample pieces produced on the developed fabrication setup

5. Conclusion

This paper has described a powerful approach and technique to materialize masonry-like vaults. A strategy for generating individual and geometrically complex voussoirs has been developed, taking into account architectural, structural and fabrication requirements. The closer investigation of these requirements resulted in a set of geometrical, interdependent constraints for the process of materialization. A customized machine setup was developed in respect to the technological, economical, and ecological relevant fabrication constraints. The setup serves as a physical testing environment to explore the feasibility of the concept. First foam testing pieces in model scale could be produced successfully to form freeform masonry-like vaults.

The promising results unveil new possibilities in real stone cutting of freeform masonry-like vaults, highlighting the feasibility and efficiency of compression-only structures. Further research will be necessary to develop strategies to generate vault tessellations based on structural requirements and appearance combined. This paper has focused on cutting geometries for masonry-like vaults, but hints at exciting possibilities for the generation of e.g. formwork for complex concrete structures or building parts. Also, lighter porous materials like foam could be used to form geometries which are then infiltrated with the appropriate material to provide the compression stiffness for vaulted structures.

6. References

[1] EVANS R., *The Projective Cast*, Cambridge, MIT Press, 1995.

[2] FALLACARA G., “Toward a Stereotomic Design: Experimental Constructions and Didactic Experiences”, *Proceedings of the Third International Congress on Construction History*, 2009.

[3] BLOCK P., and OCHSENDORF J., “Thrust Network Analysis: A new methodology for three – dimensional equilibrium”, Journal of the International Association for Shell and Spatial Structures, Vol. 48, No. 3, 2007, pp. 167-173.

[4] BLOCK P., “Thrust Network Analysis: Exploring Three-dimensional Equilibrium”, PhD dissertation. Massachusetts Institute of Technology, Cambridge, MA, USA, 2009.

[5] LACHAUER L., RIPPMANN M., and BLOCK P., “Form Finding to Fabrication: A digital design process for masonry vaults”, *Proceedings of the International Association for Shell and Spatial Structures (IASS) Symposium 2010*, Shanghai, China.

[6] BLOCK P., and LACHAUER L., “Closest-Fit, Compression-Only Solutions for Freeform Shells”, *Proceedings of the IABSE-IASS Symposium 2011*, London, UK, 2011.

[7] GARRIDO CAMPOS J., “Circular Sawblade Stone Cutting Technology Based on STEP-NC”. *Advanced Design and Manufacturing Based on STEP, Springer Series in Advanced Manufacturing*, 2009, pp. 125-143.

[8] ASCHE J., “Tiefschleifen von Granit”, PhD thesis, University of Hannover, 2000.

[9] FALLACARA G., “Digital Stereotomy and Topological Transformations”, *Proceedings of the Second International Congress on Construction History*, Vol. 1, 2006, pp. 1075-1092.

[10] FLÖRY S., and POTTMANN H., “Ruled Surfaces for Rationalization and Design in Architecture”, *Proceedings of the Conference of the Association for Computer Aided Design in Architecture (ACADIA)*, 2010.

[11] FLÖRY S., “Constrained Matching of Point Clouds and Surfaces”, PhD thesis, Institute of Discrete Mathematics and Geometry, Vienna University of Technology, 2009.

[12] YAN H. S., and LEE R. S., and YANG Y. C., "An algorithm for surface design and tool path generation in wire-cut electrical discharge machining", *International Journal of Machine Tools and Manufacture*, Vol. 35, No. 12, 1995, pp. 1703-1714.

[13] CHONGLIN L. and DONALD E., “Solid modeling of 4-axis wire EDM cut geometry”, *Computer-Aided Design*, Vol. 29, No. 12, 1997, pp. 803-810.

[14] M. JOUANEH , M. HAMMAD A. and DATSERIS P., “A flexible automated foam cutting system”, *International Journal of Machine Tools and Manufacture*, Vol. 37, No. 4, 1997, pp. 437-449.

[15] The NIST RS274NGC Interpreter - Version 3, 2000, http://linuxcnc.org/handbook/RS274NGC_3/RS274NGC_3TOC.html, 2011-04-15

[16] McNeel R. Rhino.NET Framework SDK (Software Development Kit). http://wiki.mcneel.com/developer/dotnetplugins.